Memo: **Comparison of Mertens (2011) and Westelius (2009)**
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My paper "Managing Beliefs about Monetary Policy under Discretion" (Mertens, 2011) and Westelius (2009) study different notions of "discretionary" monetary policy in the same New Keynesian model with stochastic output targets. This note briefly compares both approaches and their results. As will be seen, a crucial difference between both approaches is that monetary policy follows a rule in Westelius (2009), corresponding to the targeting rule, that would yield the optimal time-consistent policy under full information. In contrast, Mertens (2011) derives the optimal time-consistent policy under hidden information.

The intention of this note is to highlight some of the substantial differences between both approaches. There is nothing wrong with the analysis of Westelius (2009). If anything, his policy might better be characterized as following a particular rule, without necessarily satisfying an optimality condition (under imperfect information). Depending on the underlying optimization problem — which is not specified by Westelius (2009) — it also remains to be seen in which sense the rule could be time consistent.

A Model with Stochastic Output Targets

The economy is described by the New Keynesian Phillips Curve

$$\pi_t = \beta \pi_{t+1|t} + \kappa\, x_t \tag{1}$$

where π_t is inflation and x_t is the output gap. The policymaker chooses the output gap in order to minimize the present value of expected losses.

$$E_t \sum_{k=0}^{\infty} \beta^k \left\{ \alpha_p\, \pi_{t+k}^2 + \alpha_x (x_{t+k} - \bar{x}_{t+k})^2 \right\} \tag{2}$$

The expectations operator E_t and $\pi_{t+1|t}$ reflects the policymaker's and public's information sets, respectively, which will be described later. The output gap is stochastic, $\bar{x}_t$, with a persistent and a transitory component:

$$\bar{x}_t = \tau_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{3}$$

$$\tau_{t+1} = \rho\, \tau_t + \eta_{t+1} \qquad \eta_t \sim N(0, \sigma_\eta^2) \quad \text{and} \quad 0 < |\rho| < 1 \tag{4}$$

The policymaker has full information and observes the levels of both target components individually. The private sector can however not observe the realization of target shocks. Otherwise, the public has no structural uncertainty about the economy; all parameters are known, including the specification of the target process. Westelius (2009) and Mertens (2011) specify different conditioning sets for the public's signal extraction problem, which will be described next.

Imperfect Information

Both models assume imperfect information, where the public receives only one signal, and cannot disentangle the two target components. In my model, public expectations — called "beliefs" — can only condition on the observed history of policies, denoted x^t.[1] The level of the output target cannot be observed, and the public does not know the levels of τ_t and ε_t either.

In contrast, Westelius (2009) assumes that the level of the target, $\bar{x}_t$ is directly observable.[2] In his case, the public solves thus a Kalman filter with *exogenous* information

$$\bar{\tau}_{t+1|t} = \rho(1 - K_\tau)\tau_{t|t-1} + K_\tau \bar{x}_t \tag{5}$$

which can be solved independently of the equilibrium outcomes for output and inflation.

A key difference with respect is thus the *endogeneity* of information in my setting, where equilibrium is characterized as the fixed-point between optimal policy choices and a public belief system — encoded by the Kalman filter — which has to be consistent with the equilibrium choices of the policymaker.

Optimal Monetary Policy in Mertens (2011)

My paper solves for the optimal, time-consistent policy in a Markov-perfect equilibrium. The optimal policy has the form

$$x_t = f_\tau \tau_t + f_\varepsilon \varepsilon_t + f_b \tau_{t|t-1} \tag{6}$$

for some scalars f_τ, f_ε and f_b, which can be determined numerically by the fixed-point procedure described in the technical appendix of Mertens (2011). The optimal coefficients solve the first-order conditions of the underlying optimization problem, corresponding to the targeting rule:

$$\alpha_x(x_t - \bar{x}_t) + (\kappa\,\alpha_p)\,\pi_t + \rho K_\tau \underbrace{\left((\beta\, g\,\alpha_p)\,\pi_t + v_{33}\tau_{t+1|t} + v_{13}\tau_t\right)}_{\mu_t} = 0 \tag{7}$$

where μ_t summarizes the effects of policy on public beliefs. Furthermore, K_τ is the Kalman gain used by the public in updating its beliefs $\tau_{t|t}$ in response to x_t, v_{33} and v_{13} are coefficients of the policymaker's value function and g embodies the public's perception about the dependence of future inflation on future targets: $\pi_{t+1|t} + g\tau_{t+1|t}$. While the policymaker takes these coefficients as given in his optimization, equilibrium requires that they are consistent with the optimal policy coefficients chosen by the policymaker.[3]

[1] In principle, this includes also the history of inflation rates π^t. But as a choice variable of the private sector, inflation merely reflects the private sectors information set, without providing additional information beyond x^t.

[2] As will be seen momentarily, he then needs to assume that monetary policy does not provide any additional information about the target components. Otherwise, with two unknowns and two observables, the public could perfectly infer the value of both components.

[3] As discussed in my paper, μ_t corresponds to the multiplier on the belief constraint in the policymaker's optimization problem. This multiplier captures two effects: First, the direct effect of beliefs on current outcomes via the Phillips Curve (the term in $g\pi_t$). Second, the effect of beliefs on future outcomes.

Monetary Policy in Westelius (2009)

In contrast, Westelius (2009) stipulates that policy follows the targeting rule, which would be optimal under full information, that is (7) when $\mu_t = 0$:

$$\alpha_x(x_t - \bar{x}_t) + (\kappa\,\alpha_p)\,\pi_t = 0 \tag{8}$$

Equilibrium is characterized by this targeting rule, together with the Phillips Curve (1) and the Kalman filter (5).

It is straightforward to show that the output gap process in Westelius (2009) has the same *form* as (6), however with different coefficients. In particular, Westelius' model imposes $f_\tau = f_\varepsilon$. To see this notice that targeting rule (8) and Phillips Curve (1) can be combined into

$$\pi_t = \tilde{\kappa}\bar{x}_t + \tilde{\beta}\pi_{t+1|t} \qquad \text{where} \quad \tilde{\kappa} = \kappa \cdot \frac{\alpha_x}{\alpha_x + \alpha_p\kappa^2} \qquad \text{and} \quad \tilde{\beta} = \beta \cdot \frac{\alpha_x}{\alpha_x + \alpha_p\kappa^2}$$

From $\bar{x}_{t+1|t} = \tau_{t+1|t}$ and the Phillips Curve follows

$$\pi_{t+1|t} = \frac{\tilde{\kappa}}{1 - \tilde{\beta}\rho}\,\tau_{t+1|t} \qquad\qquad \pi_t = \tilde{\kappa}\bar{x}_t + \frac{\tilde{\kappa}}{1 - \tilde{\beta}\rho}\,\tau_{t+1|t}$$

and we obtain outcomes of the form

$$\pi_t = g_{\bar{x}}\bar{x}_t + g_b\tau_{t|t-1} \qquad\qquad x_t = f_{\bar{x}}\bar{x}_t + f_b\tau_{t|t-1}$$

where the coefficients $g_{\bar{x}}$, g_b, $f_{\bar{x}}$, and f_b can be determined by straightforward substitution from the Kalman filter (5) and the targeting rule (8).

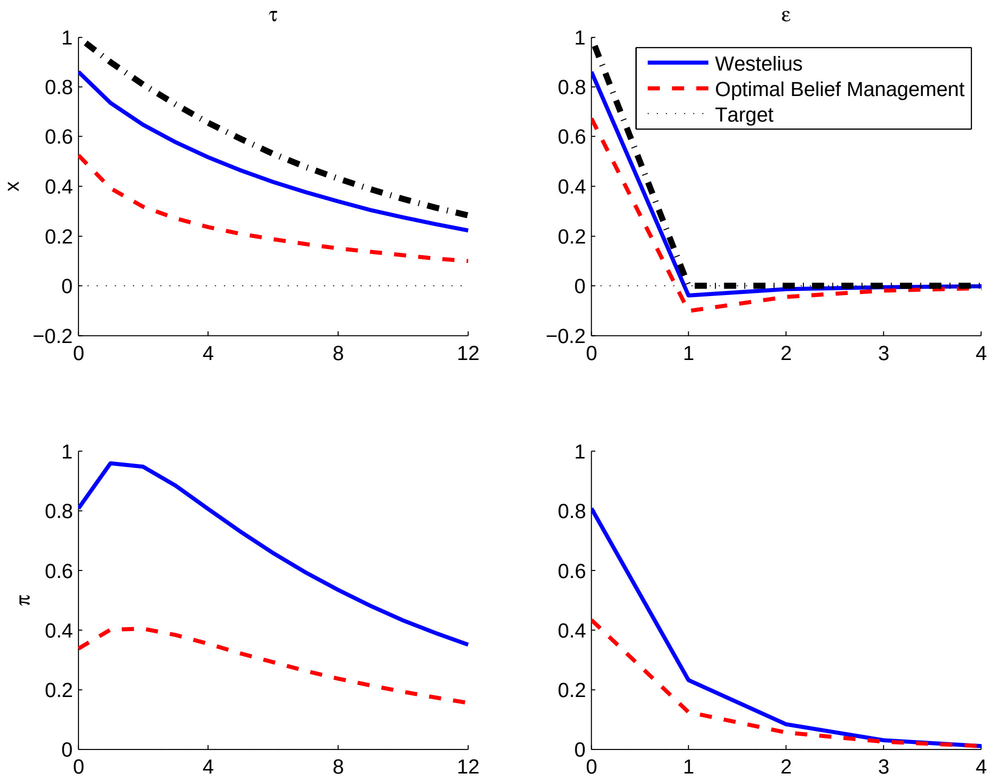
Observing the output gap, does not reveal any information beyond what is known from observing the history of target levels ($\bar{x}^t$), since policy reacts identically to incoming shocks without discriminating between the two target components, $f_{\bar{x}} = f_\tau = f_\varepsilon$. Otherwise – with two shocks and two observables — there would be complete revelation and the full information outcome would be achieved.

Differences in Outcomes

Figure 1 compares the outcomes under both policies. As can be seen from the top panels, the policy of Westelius (2009) reacts identically to incoming shocks, causing an identical effect of inflation on impact. In contrast, the optimal belief policy of Mertens (2011) distinguishes between the different sources of the target process. More importantly, the disciplinary channel of belief management (discussed in my paper) induces the policymaker to scale back his pursuit of either target, since he recognizes the inflationary effect of such policies on public beliefs (at least as much as possible while retaining time consistency).

The policy of Westelius (2009) is based on a rule, and not an explicit time-consistent optimization problem which takes into account the hidden information setting. As shown in Figure 1, the optimal discretion policy (Mertens, 2011) achieves lower inflation, while missing the output targets to a larger extent than the rule of Westelius (2009). In terms of policy losses, the effects can

Figure 1: Optimal Belief Management versus Targeting Rule from Full Info



Note: Baseline calibration as documented in Mertens (2011). All shocks have unit variance.

thus be ambiguous, but at least for the calibration used in Figure 1, the optimal discretion policy leads to lower losses, since the improvements in inflation far outweigh the worsening in the output losses. In terms of "representative" welfare, as measured by $\alpha_p \pi_t^2 + \alpha_x x_t^2$, the optimal discretion policy is however unambiguously better in this case.

References

Mertens, E. (2011, February). Managing beliefs about monetary policy under discretion. *mimeo*, Federal Reserve Board.

Westelius, N. J. (2009, April). Imperfect transparency and shifts in the central bank's output gap target. *Journal of Economic Dynamics and Control 33*(4), 985–996.