Forecasting with Shadow-Rate VARs

Andrea Carriero[1] Todd E. Clark[2]
Massimiliano Marcellino[3] Elmar Mertens[4]

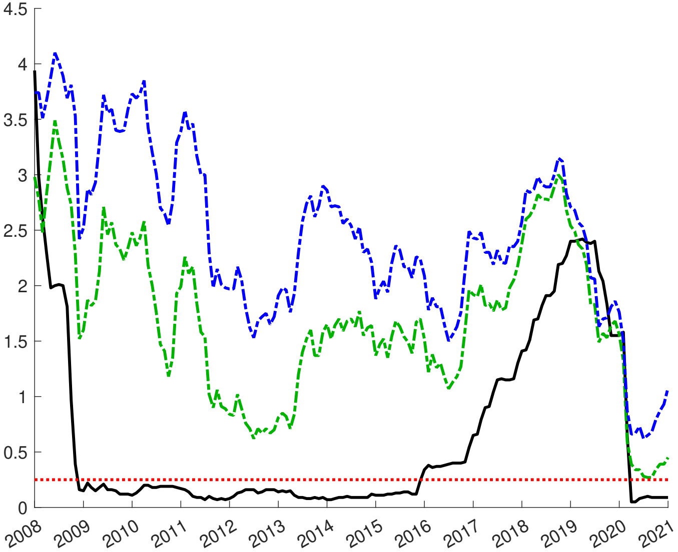
[1]Queen Mary University of London, [2]Federal Reserve Bank of Cleveland,
[3]Bocconi University, IGIER and CEPR, [4]Deutsche Bundesbank

March 2021

NOMINAL INTEREST RATES SINCE 2008

U.S.

Federal funds rate (black), 5-year Treasury (green), 10-year Treasury (blue)



Since early 2020: Nominal interest rates are back at the ELB

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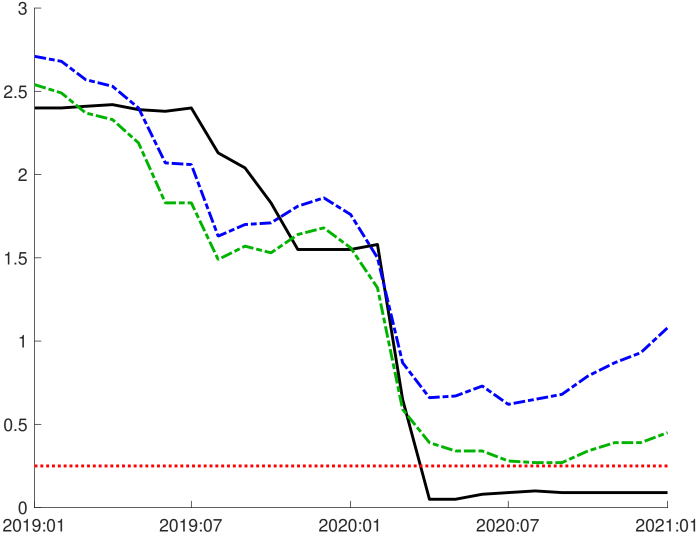


Even longer-term bonds are now at/near the ELB

VARS AND ELB DATA

VARs are a great forecasting tool . . .

- Successful track record in . . .

  point and density forecasting
  structural analysis

- Well-documented benefits of

  parameter shrinkage via priors,
  modeling of shocks with time-varying volatility

. . . but ill-equipped to handle bounded data

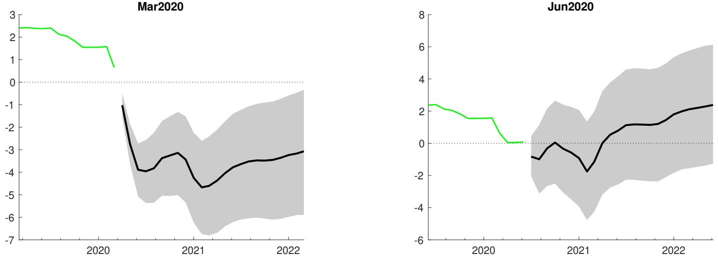
Standard VAR is linear:

$$\boldsymbol{y_t = \Pi_0 + \Pi(L) y_{t-1} + v_t\,, \quad v_t \sim N(0, \Sigma_t)}$$

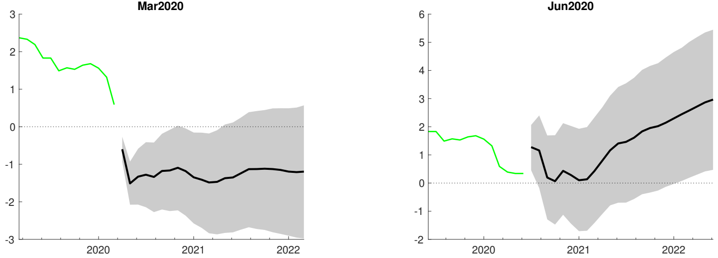
VAR FORECASTS FOR INTEREST RATES IN 2020

homoscedastic BVAR with 18 variables

Federal funds rate



5-year Treasury yield



WAYS TO ACCOMODATE ELB

- Use longer-term yields rather than short-term policy rates (Swanson & Williams, 2014; Debortoli et al, 2019)

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- Plug-in approach: External shadow-rate estimates as data

  Inconsistent estimates and generated regressor (Mavroeidis, 2020; Krippner, 2020; Aruoba, et al 2021)

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Our approach: censored data in a VAR

"Shadow rate" as latent process, that is a useful state variable for forecasting

RELATED LITERATURE

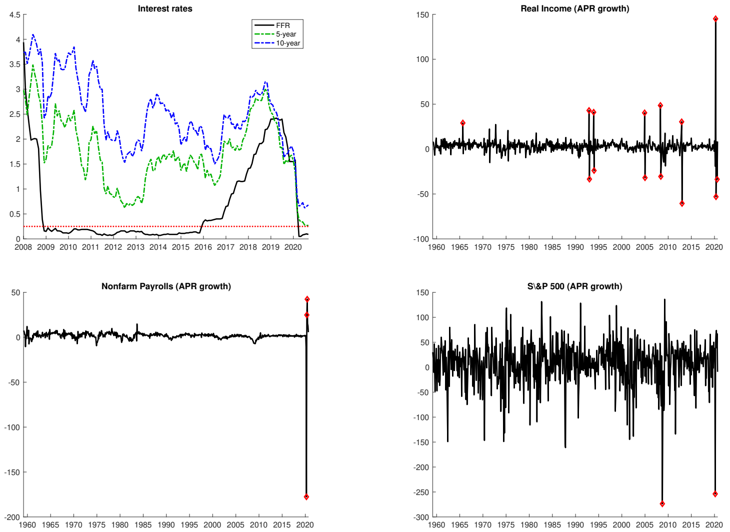
No-arbitrage term-structure models (w/shadow rates)

- Black (1995)
- Krippner (2013/15/20), Wu & Xia (2016/20)
- Bauer & Rudebusch (2014/17), Kim & Singleton (2011), Priebsch (2017), Christensen & Rudebusch (2015)
- Joslin et al. (2013), Christensen et al (2009/11)

Time-series models

- Johannsen & Mertens (2021), Gonzalez-Astudillo and Laforte (2020)
- Mavroeidis (2020), Aruoba, Mlikota, Schorfheide, Villalvazo (2021)
- Iwata & Wu (2006), Nakajima (2011), Chan & Strachan (2014), Bäurle et al (2016)

OTHER CHALLENGES IN 2020 DATA



Focus today: Nominal interest rates at the ELB.

We consider COVID-related outliers in companion work

AGENDA

1. **Shadow-rate concept**
2. **Shadow-rate estimates**
3. **Shadow-rate vs. missing-data approach**
4. **Interest-rate predictions**
5. **Forecast comparison for macro and financial variables**
6. **Conclusion**

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THE SHADOW RATE OF INTEREST

Observed Rate i_t

$$i_t = \max\left(s_t, ELB\right)$$

Shadow Rate s_t

Nominal interest rate that would prevail in the absence of lower bound constraint

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Model s_t with typical time-series tools
(w/o no-arbitrage restrictions)
and handle $\max$ operator in measurement equation

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Our paper

- applies Johannsen & Mertens (2021) to general VAR
- more efficient shadow-rate sampling

SHADOW-RATE VAR

All expressed in companion form omitting intercepts

Setup

- Partition the VAR vector:

$$\boldsymbol{y_t} = \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{i_t} \end{bmatrix}$$

- Define a corresponding shadow-rate VAR vector

$$\boldsymbol{z_t} = \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{s_t} \end{bmatrix}, \qquad \boldsymbol{i_t = \max\left(s_t, ELB\right)}$$

Shadow-rate VAR

$$\boldsymbol{z_t = \Pi\, z_{t-1} + v_t} \qquad\qquad \boldsymbol{v_t \sim N(0, \Sigma_t)}$$

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MCMC sampling of shadow-rate VAR

- For given shadow rates, we know how to draw $\boldsymbol{\Pi}$ and $\boldsymbol{\Sigma_t}$
- We treat $\boldsymbol{ELB}$ as known value of 25bp (or 12.5bp)
- Additional step: draw shadow rates *consistent with ELB*

ONGOING WORK: HYBRID VAR

Include lags of actual and shadow interest rates

$$z_t = \Pi \, z_{t-1} + F \, (i_{t-1} - s_{t-1}) + v_t$$

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equivalently:

$$\boldsymbol{z_t = \Pi_x x_{t-1} + \underbrace{(\Pi_s - F)}_{\Pi_s^*}\ s_{t-1} + F\, i_{t-1} + v_t}$$

with $\boldsymbol{\Pi\, z_{t-1} = \Pi_x\, x_{t-1} + \Pi_s\, s_{t-1}}$

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Identification challenge

- Recall: away from ELB we have $i_t = s_t$
- Π can be identified pre ELB, but F needs ELB data
- But, at ELB, the regressor for F, $i_{t-1} - s_{t-1}$, is latent

To be considered: ELB interaction dummies

$$\mathbb{1}\,(i_{t-1} \cdot ELB) \text{ on } x_{t-1} \text{ and } i_{t-1}$$

SHADOW-RATE SAMPLING

Additional step within MCMC sampler for VAR, given Π, Σ_t, $\forall\, t$

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$$\boldsymbol{y_t} = \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{i_t} \end{bmatrix}, \qquad \boldsymbol{z_t} = \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{s_t} \end{bmatrix}, \qquad \boldsymbol{i_t} = \max\left(\boldsymbol{s_t}, \boldsymbol{ELB}\right)$$

- $\boldsymbol{Y}$, vector of all $\boldsymbol{y_t}$
- $\bar{\boldsymbol{Y}}$, all of $\boldsymbol{Y}$ except for $\boldsymbol{i_t} = \boldsymbol{ELB}$
- $\boldsymbol{S}$, all shadow rates $\boldsymbol{s_t}$ when the ELB binds

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Missing-value problem

$$\boldsymbol{S}|\bar{\boldsymbol{Y}} \sim \boldsymbol{N}(\boldsymbol{\mu}, \boldsymbol{\Omega})$$

can be obtained from standard Kalman smoothing

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Shadow-rate sampling problem

$$\boldsymbol{S}|\boldsymbol{Y} \sim \text{truncN}\left(\boldsymbol{\mu}, \boldsymbol{\Omega}, \boldsymbol{S} \leq \boldsymbol{ELB}\right)$$

- Johannsen & Mertens (2021): Rejection sampling
- Here: Direct sampling from truncated multivariate normal

DETAILS ABOUT SHADOW-RATE SAMPLING

BACKUP

Problem: $S|Y \sim \text{truncN}\left(\mu, \Omega, S \leq ELB\right)$

- μ and Ω implied by VAR and $\bar{Y}$
- With T^* obs at the ELB, S is large ($T^* \cdot N_s$)
- But Ω is sparse, and μ, Ω can be computed recursively
- $S \leq ELB$ holds elements-wise for entire trajectory

Implementation

- Johannsen & Mertens (2021): Rejection sampling. Worked there, but too many rejections here
- We do Gibbs sampling from multivariate truncN (Geweke, 1991):

$$s_t | s_{1:t-1}, s_{t+1:T}, \bar{Y}$$

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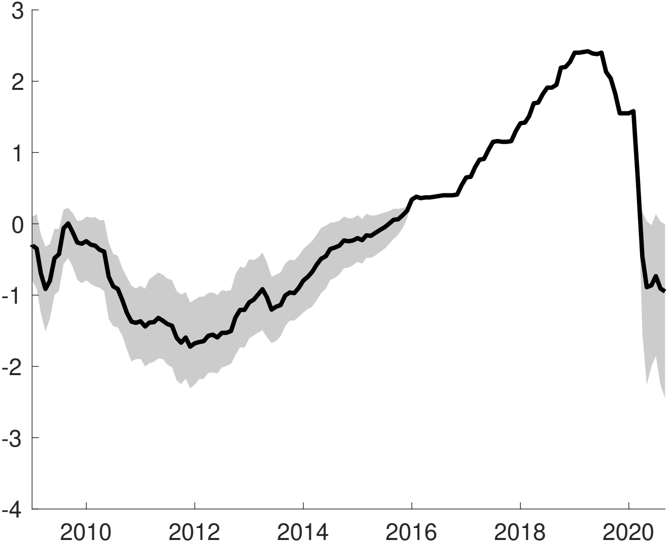
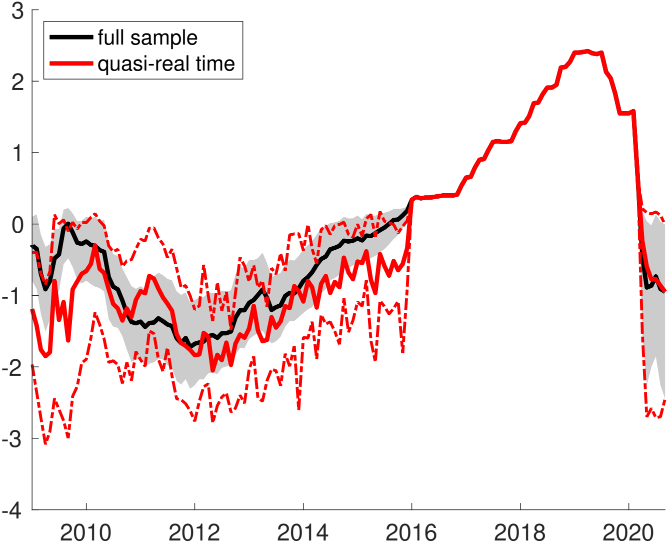
SHADOW-RATE ESTIMATES
median and 90% bands



Corresponds to VAR-implied FFR “prescriptions” (w/o ELB)

SHADOW-RATE ESTIMATES

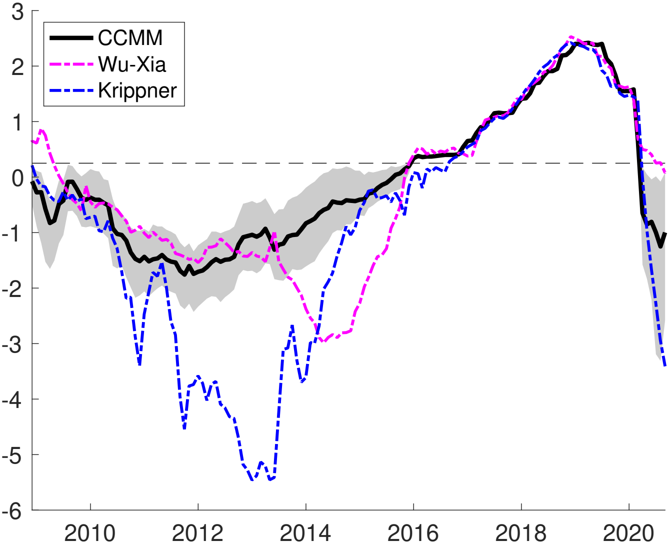
medians and 90% bands



decent quasi-real time properties

OTHER SHADOW-RATE ESTIMATES

CCMM (black): median and 90% bands



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INFORMATION CONTENT OF BINDING ELB FOR VAR

Let's consider the following thought experiments

Let's record . . .

1. Shadow-rate draws $S|Y$, from shadow-rate VAR

Purpose:

1. This is our baseline

INFORMATION CONTENT OF BINDING ELB FOR VAR

Let's consider the following thought experiments

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2. Missing-data draws, $S|\bar{Y}$, from shadow-rate VAR

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2. Tells us to what extent ELB is binding for the sampler *at the parameters estimated with shadow-rate VAR*

INFORMATION CONTENT OF BINDING ELB FOR VAR

Let's consider the following thought experiments

Let's record . . .

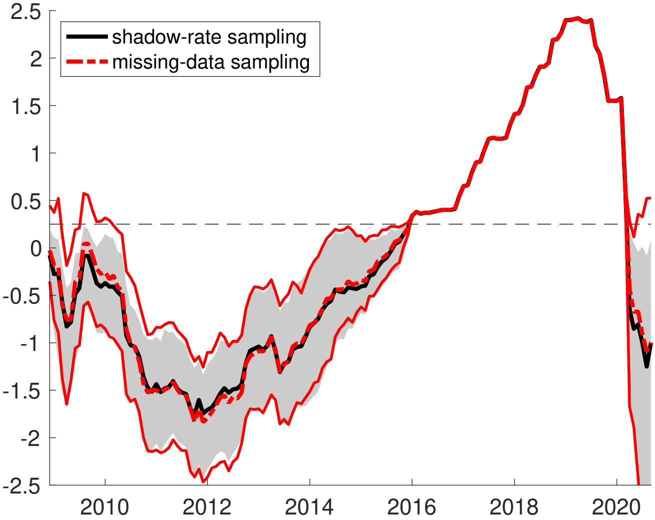
1. Shadow-rate draws $S|Y$, from shadow-rate VAR
2. Missing-data draws, $S|\bar{Y}$, from shadow-rate VAR
3. Missing-data draws, $S|Y$, from a missing-data VAR estimated on $\bar{Y}$ rather than Y (e.g. Del Negro et al., BPEA, 2017)

Purpose:

1. This is our baseline
2. Tells us to what extent ELB is binding for the sampler *at the parameters estimated with shadow-rate VAR*
3. Shows us if ELB sampling shifted VAR parameters

EFFECT OF CONDITIONING ON ELB

Cases 1&2: $S|Y$ (black) vs $S|\bar{Y}$ (red) with $\Pi, \Sigma_t|Y$, median/90% bnds.

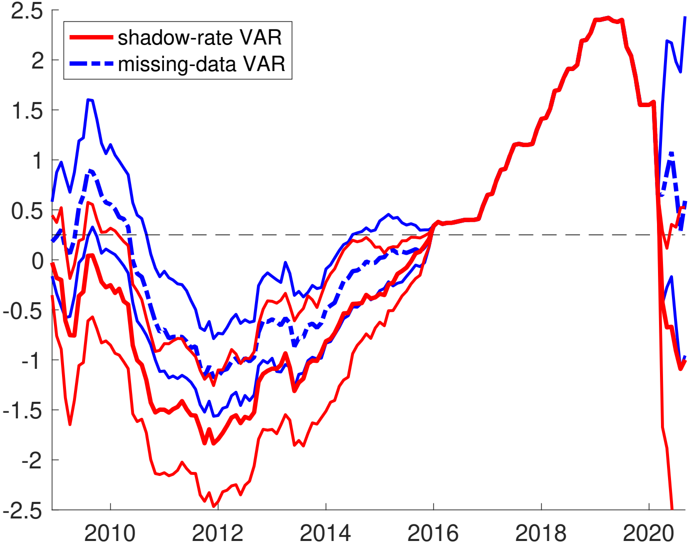


truncation appears negligible

at parameters from shadow-rate VAR posterior

EFFECT OF CONDITIONING ON ELB

Cases 2&3: $S|\bar{Y}$ from $\Pi, \Sigma_t|\bar{Y}$ (blue) or $\Pi, \Sigma_t|Y$ (red), median/90% bnds.



Missing-data VAR parameters see $S|\bar{Y} > ELB$ more often

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FORECAST EVALUATION

All models with stochastic volatility (SV) in VAR residuals

Competing forecasts

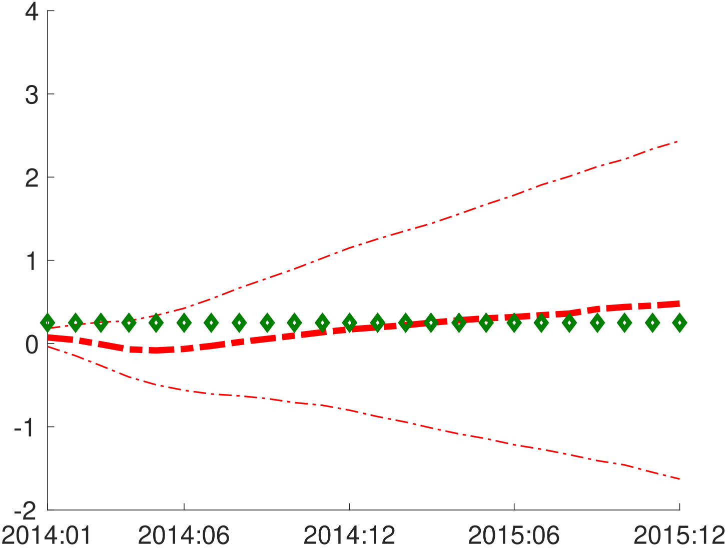
- **Standard VAR**
- **Shadow-rate VAR**
- **Truncated VAR**: nominal-rate predictions are censored during forecast simulation with standard VAR
- **Plug-in VAR**: standard VAR with WuXia/Krippner rates, censoring applied *after* simulation of VAR forecasts

QUASI-REAL TIME SETUP

- All data from same FRED-MD vintage
- 18 macro- and financial variables
- Monthly observations since 1959:03
- Growing estimation windows
- Evaluation window 2009:01 – 2020:09 (similar results through 2017:12)
- Forecasts up to two years out ($\boldsymbol{h = 24}$)

STANDARD FUNDS-RATE PREDICTIONS

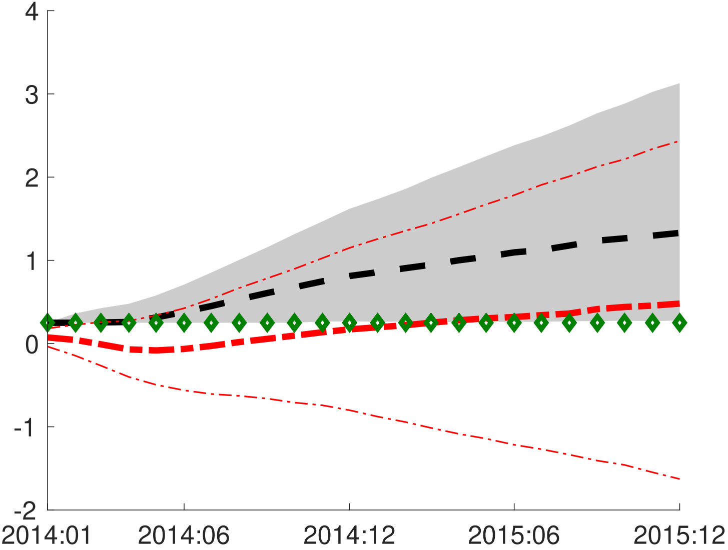
2013:12



Median and 68% bands of predictive densities. Realized (diamonds).

STANDARD FUNDS-RATE PREDICTIONS

Standard VAR: regular (red) vs truncated (black) densities



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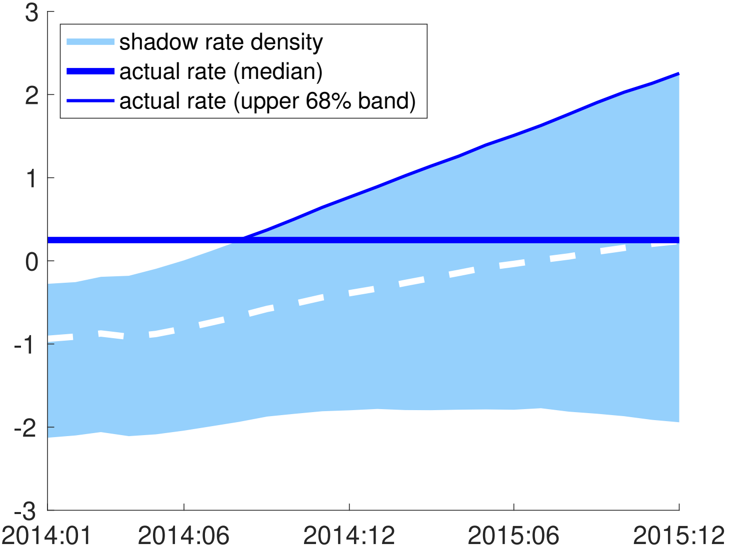
SHADOW-RATE VAR PREDICTIONS

Shadow-rate (light blue) and actual-rate (dark blue) densities



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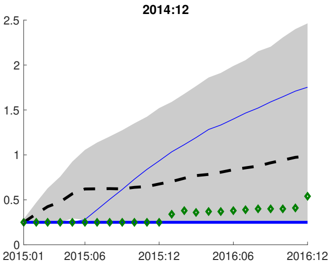
FFR PREDICTIONS IN THE 2010s
Truncated (black), vs shadow-rate VAR (blue)



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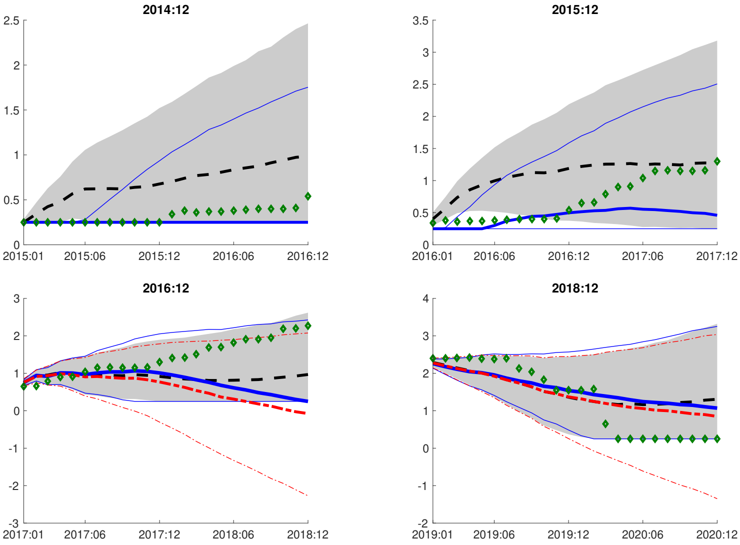
FFR PREDICTIONS IN THE 2010s

Standard (red), truncated (black), vs shadow-rate VAR (blue)



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Forecast evaluation metrics

- Mean forecasts: root-mean squared errors (RMSE)
- Median forecasts: median absolute deviations (MAD)
- Densities: Continuously-ranked probability score (CRPS)
- Significance with Diebold-Mariano-West (DMW) tests

18-VARIABLE DATA SET

BACKUP

Monthly obs from 1959:M03 to 2020:M09; FRED-MD vintage 2020:10

Variable	FRED-MD code	Transformation
Real Income	RPI	$\Delta \log(x_t) \cdot 1200$
Real Consumption Exp.	DPCERA3M086SBEA	$\Delta \log(x_t) \cdot 1200$
IP	INDPRO	$\Delta \log(x_t) \cdot 1200$
Capacity Utilization	CUMFNS	
Unemployment Rate	UNRATE	
Nonfarm payrolls	PAYEMS	$\Delta \log(x_t) \cdot 1200$
Hours	CES0600000007	
Hourly Earnings	CES0600000008	$\Delta \log(x_t) \cdot 1200$
PPI: Finished Goods	WPSFD49207	$\Delta \log(x_t) \cdot 1200$
PPI (Metals)	PPICMM	$\Delta \log(x_t) \cdot 1200$
PCE prices	PCEPI	$\Delta \log(x_t) \cdot 1200$
Federal Funds Rate	FEDFUNDS	
Housing Starts	HOUST	$\log(x_t)$
SP500	SP500	$\Delta \log(x_t) \cdot 1200$
U.S. / U.K. Forex	EXUSUKx	$\Delta \log(x_t) \cdot 1200$
5-Year yield	GS5	
10-Year yield	GS10	
Baa spread	BAAFFM	

SHADOW-RATE VARS

RELATIVE RMSE

Values below one indicate improvement over Standard VAR

	Truncated			Shadow rate		
Variable / Horizon	3	12	24	3	12	24
Income	1.00	1.00	1.00			
Consumption	1.00	1.00	1.00			
IP	0.99	1.01	0.99*			
Cap. Util.	1.00	1.16	1.30			
Unemp.	1.00	1.01**	1.11**			
Nfm Pyrlls	1.00	1.00	1.00			
Hours	1.01	1.13	1.35			
H. Earnings	1.00	0.97	0.92**			
PPI (Fin.)	1.00	0.97	0.94			
PPI (Metals)	1.01	0.99	1.01			
PCE Prices	0.98*	0.90	0.77			
FFR	0.46*	0.58	1.02			
Hsng Strts	1.03	1.00	1.08			
S&P 500	1.01	1.03	1.04			
USD / GBP	1.01	1.00	1.01			
5y Treas	1.01	1.29	1.93			
10y Treas	1.05	1.34	2.27			
Baa	0.96	1.25	2.26			

Note: Stars denote DMW significance. Eval from 2009:01 through 2020:09.

SHADOW-RATE VARS

RELATIVE RMSE

Values below one indicate improvement over Standard VAR

	Truncated			Shadow rate		
Variable / Horizon	3	12	24	3	12	24
Income	1.00	1.00	1.00	1.00	1.00	0.99
Consumption	1.00	1.00	1.00	1.00	1.00	1.00
IP	0.99	1.01	0.99*	1.00	1.01	1.01
Cap. Util.	1.00	1.16	1.30	1.01	1.08	1.30***
Unemp.	1.00	1.01**	1.11**	1.00	0.99	1.00
Nfm Pyrlls	1.00	1.00	1.00	1.00	1.00	1.00
Hours	1.01	1.13	1.35	1.06	1.00	1.08
H. Earnings	1.00	0.97	0.92**	0.99	1.00	0.96***
PPI (Fin.)	1.00	0.97	0.94	1.02	1.03	0.99
PPI (Metals)	1.01	0.99	1.01	1.01	1.01	0.99**
PCE Prices	0.98*	0.90	0.77	1.06**	1.09	1.06
FFR	0.46*	0.58	1.02	0.36*	0.34	0.52**
Hsng Strts	1.03	1.00	1.08	1.05	1.04	0.95
S&P 500	1.01	1.03	1.04	0.98*	0.99	0.99
USD / GBP	1.01	1.00	1.01	0.99	0.99	0.97***
5y Treas	1.01	1.29	1.93	0.93	0.77**	0.70**
10y Treas	1.05	1.34	2.27	0.98	0.81	0.64**
Baa	0.96	1.25	2.26	1.00	0.89	0.83

Note: Stars denote DMW significance. Eval from 2009:01 through 2020:09.

SHADOW-RATE VARS

RELATIVE MAD

Values below one indicate improvement over Standard VAR

	Truncated			Shadow rate		
Variable / Horizon	3	12	24	3	12	24
Income	1.06	1.15	1.03	1.00	0.99	1.07
Consumption	1.05	1.06	0.89	1.06	1.11	0.95
IP	0.90	1.09	0.85	1.06	1.15	1.10
Cap. Util.	1.04	1.09	1.15	1.16	1.39	1.92
Unemp.	1.05	1.20	1.39	1.08	1.13	1.00
Nfm Pyrlls	1.02	1.19	1.32	1.05	1.30	1.30
Hours	1.00	0.99	1.37	1.02	1.06	1.29
H. Earnings	1.01	0.80	0.87	1.05	1.10	0.99
PPI (Fin.)	1.03	0.99	0.91	1.06	0.94	0.89
PPI (Metals)	1.00	1.00	1.02	0.99	0.98	1.07
PCE Prices	1.03	0.86	0.85	1.07	1.02	1.08
FFR	0.76	1.11	1.10	0.00	0.00	0.00
Hsng Strts	0.98	0.84	1.44	0.95	0.87	0.84
S&P 500	1.01	1.02	1.08	0.97	0.95	1.07
USD / GBP	1.01	1.00	1.04	0.97	0.97	0.96
5y Treas	1.02	1.42	1.09	0.81	0.76	0.60
10y Treas	0.97	1.42	1.40	0.96	0.88	0.82
Baa	1.01	0.84	0.86	1.07	0.96	0.96

Note: Stars denote DMW significance. Eval from 2009:01 through 2020:09.

SHADOW-RATE VARS

RELATIVE CRPS

Values below one indicate improvement over Standard VAR

Variable / Horizon	Truncated 3	Truncated 12	Truncated 24	Shadow rate 3	Shadow rate 12	Shadow rate 24
Income	1.00	1.01	1.01	0.99	1.00	1.01
Consumption	1.00	1.00	1.00	1.00	1.02**	1.02
IP	1.00	1.01	0.99	1.01**	1.04**	1.07**
Cap. Util.	1.01	1.10	1.15**	1.04***	1.12***	1.24***
Unemp.	1.00	1.05***	1.18***	1.01*	1.02	1.03
Nfm Pyrlls	1.00	1.03*	1.07***	1.00	1.02**	1.05***
Hours	1.00	1.11*	1.22**	1.04**	1.05	1.13**
H. Earnings	1.00	0.99	0.97***	1.00	1.00	1.00
PPI (Fin.)	1.00	0.99	0.98	1.01	1.02	1.00
PPI (Metals)	1.00	1.00	1.01	1.00	1.00	1.01*
PCE Prices	0.99**	0.95	0.90*	1.04	1.06	1.04
FFR	0.47**	0.62	0.70	0.28***	0.29**	0.34***
Hsng Strts	1.01	1.02	1.12	1.02	0.96	0.90
S&P 500	1.01*	1.02	1.02	0.99	1.00	1.02***
USD / GBP	1.01	1.00	1.02	0.99	0.99	1.00
5y Treas	0.99	1.20	1.40	0.92	0.81**	0.69***
10y Treas	1.01	1.24*	1.60	0.96	0.86	0.80**
Baa	0.99	1.06	1.21	1.01	0.96	1.00

Note: Stars denote DMW significance. Eval from 2009:01 through 2020:09.

COMPARISON AGAINST PLUG-IN SHADOW RATES

Shadow-rate VAR vs plug-in approach with Wu-Xia rates (in denominator)

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	1.00	1.00	0.98***	1.00	0.98	0.99	1.00	0.99	1.00
Consumption	1.00	1.00	1.00	0.98	1.00	0.99	1.00	1.00	1.00
IP	1.00	1.00	1.00	1.01	1.02	1.00	1.01	1.01	1.01
Cap. Util.	0.97	1.00	1.03	0.98	1.00	1.07	0.98	0.99	1.01
Unemp.	1.00	1.00	1.04	0.99	0.93**	1.06	0.99**	0.96**	1.00
Nfm Pyrlls	1.00	1.00	1.00	0.98**	0.99	1.05	0.99**	0.99	1.01
Hours	0.98*	0.95	1.02	0.96***	0.93	1.01	0.98*	0.96	1.05
H. Earnings	0.99	0.99	0.97	0.99	0.99	0.97	0.99	0.99	1.01
PPI (Fin.)	1.02	1.01	0.98	1.01	1.02*	0.98	1.01	1.01	0.99
PPI (Metals)	1.00	1.00	0.99	1.00	1.00	0.99	1.00	1.00	1.01*
PCE Prices	1.02	1.01	0.99	1.02	1.00	0.99	1.00	1.00	0.99
Policy Rate	0.80**	0.95	1.10**	0.76**	0.85*	0.97	0.77**	0.94	1.00
Hsng Strts	1.02	1.11	1.13	0.99	0.98	0.94	0.99	1.00	1.00
S&P 500	1.01	0.99	0.99	0.98	0.99	0.99	1.00	0.99	1.01*
USD / GBP	0.99	1.00	0.98	1.01	1.00	0.99	0.99	1.00	1.00
5y Treas	1.04	1.02	0.98	1.00	0.91	0.83	1.02	0.96	0.90
10y Treas	1.01	0.95	0.79**	0.99	0.96	0.77*	1.00	0.94	0.83**
Baa	1.02	1.07	0.85	1.02	0.95	0.86	1.02	0.96	0.96

Note: Stars denote DMW significance. Eval from 2009:01 through 2020:09.

COMPARISON AGAINST PLUG-IN SHADOW RATES

Shadow-rate VAR vs plug-in approach with Krippner rates (in denominator)

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	1.00	1.00	0.98***	0.99	1.00	0.99	0.99**	0.99*	0.99
Consumption	1.00	1.00	1.00	0.99	0.98	0.96**	1.00	0.98*	0.98*
IP	1.01	1.01	0.99	1.02	1.04	0.97	1.01	1.01	0.98
Cap. Util.	0.99	1.07	1.12	1.02	1.09	1.13	1.00	1.05	1.06
Unemp.	1.00	1.00	1.06	0.99	0.90*	1.06	0.98	0.95*	1.01
Nfm Pyrlls	1.00	1.00	1.00	0.99	0.98	0.99	1.00	0.98***	0.98
Hours	1.01	0.97	1.04	0.98	0.97	1.02	0.99	0.98	1.05
H. Earnings	1.00	1.01	0.96*	0.99	0.99	0.97	0.99	1.01	1.01
PPI (Fin.)	0.99	0.97**	0.97	0.98	0.97	0.97	0.98	0.97*	0.98
PPI (Metals)	0.99	0.99	0.99	0.99	0.99	0.98	0.99	0.99	1.00
PCE Prices	1.00	0.95*	0.92*	0.99	0.96	0.94	0.98	0.95	0.95
Policy Rate	0.83*	0.89	1.02	0.83	0.85	0.97	0.84	0.89	0.99
Hsng Strts	1.01	1.08	1.01	0.97	0.95	0.86	0.98	0.96	0.90
S&P 500	1.00	1.00	0.99	0.96*	0.99	0.96***	0.98	0.99	0.99
USD / GBP	1.01	1.00	0.99	1.03*	1.02	0.99	1.01	1.00	0.99
5y Treas	1.04	1.05	0.98	1.02	0.96	0.81**	1.00	0.97	0.86**
10y Treas	0.98	0.93	0.80**	0.96	0.89	0.69**	0.97	0.90	0.78***
Baa	1.02	0.97	0.74**	0.93	0.85	0.72**	0.95	0.84	0.80***

Note: Stars denote DMW significance. Eval from 2009:01 through 2020:09.

ROBUSTNESS CHECKS

We also consider the following alternatives:

- Replaced 10-year by 20-year yield
- Use 3-month Tbill rather than Federal Funds Rate
- Set ELB to 12.5 basis points (rather than 25bp)
- Evaluation window stops in 2017:12
  (to avoid COVID-related realizations)

Without much change in results,

if anything:

shadow-rate VAR with 3m Tbill better than baseline

AGENDA

CONCLUSIONS

Standard VARs ignore the ELB

- Negative policy-rate projections
- Poor man's fix: standard estimation, then truncate simulated interest-rate densities
- Due to own persistence of policy rate: lift-off always imminent at ELB

Our solution: Shadow-rate VARs

- Internally consistent inference and better forecasts, than standard VAR (even w/truncated densities)
- . . . forecasts often better than VARs with plug-in shadow-rate estimates
- Own persistence of shadow rate consistent with policymakers tracking notional-rate prescriptions
- Scalable to multiple interest-rate maturities at ELB