Comments on
Variance of the IID estimator
in Lo (2002)

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Summary

In section 1 I offer what I regard as the necessary correction to the asymptotic variance of the Sharpe Ratios' IID estimator as presented by Lo (2002):

$$V_{IID} = 1 + \frac{1}{2} \cdot \mathrm{SR}^2 - \mathrm{SR} \cdot \gamma_3 + \mathrm{SR}^2 \cdot \frac{\gamma_4 - 3}{4}$$

This formula holds for any, possibly non-normal IID distribution. Skewness to the left ($\gamma_3 < 0$) and leptokurtosis ($\gamma_4 > 3$) make the estimator's variance bigger than in the normal case to which the formula and table 1 of Lo should only apply. For instance, using the values for skewness and kurtosis of the Fama-French factor portfolios, the variances of table 1 in Lo (2002) are up to 70% off. In the ballpark of the Fama-French portfolios' Sharpe Ratios, the numbers are only 1–10% off.

The necessary correction can also be read off from the more general formulas in Lo's discussion of time-aggregation and IID returns.

In section 2, I recap the GMM setup of the estimation in order to (re)establish the notation. Section A contains the Matlab code used to produce tables and figures reported here.

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1 Corrected Formula for Variance Estimator in IID Case

Lo (2002, Formula A1) presents a formula for the Sharpe Ratio's estimator under IID returns. That formula should only be applicable to IID *normal* returns[1]. The correct formula should be based on the following asymptotic variance of the moment estimator $\boldsymbol{\theta}$. (Please refer to section 2 for the estimation setup and notation.)

$$\begin{aligned}
\boldsymbol{V}_{\boldsymbol{\theta}} &\overset{IID}{=} E(\boldsymbol{H}_t(\boldsymbol{\theta}_0)\boldsymbol{H}_t(\boldsymbol{\theta}_0)') \\
&= E\left\{\begin{bmatrix} (R_t-\mu)^2 & (R_t-\mu)\left[(R_t-\mu)^2-\sigma^2\right] \\ (R_t-\mu)\left[(R_t-\mu)^2-\sigma^2\right] & \left[(R_t-\mu)^2-\sigma^2\right]^2 \end{bmatrix}\right\} \\
&= \begin{bmatrix} \sigma^2 & E(R_t-\mu)^3 \\ E(R_t-\mu)^3 & E(R_t-\mu)^4-\sigma^4 \end{bmatrix} \qquad (1)
\end{aligned}$$

See for example Goldberger (1964, p. 120), Theil (1973, p. 373) and Greene (2000, p. 142). In the discussion of time aggregation, Lo, Formula (A27), derives himself the corresponding elements in my equation (1) and points out that in order to obtain something like his (A1) the assumption of normality is needed[2].

Only if there is zero skewness[3] and zero excess kurtosis[4], equation (1) can be simplified to obtain formula (A1) of Lo:

$$\boldsymbol{V}_{\boldsymbol{\theta}} \overset{NIID}{=} \begin{bmatrix} \sigma^2 & 0 \\ 0 & 2\sigma^4 \end{bmatrix} \qquad \text{(Lo A1)}$$

Using (1), the variance of the estimated Sharpe Ratio equals formula (A4) of Lo *plus* two additive terms depending on skewness and excess kurtosis of the return distribution:

$$\begin{aligned}
V_{IID} &= \frac{\partial g}{\partial \boldsymbol{\theta}'} \boldsymbol{V}_{\boldsymbol{\theta}} \frac{\partial g}{\partial \boldsymbol{\theta}'}' \\
&= \begin{bmatrix} \frac{1}{\sigma} & \frac{-(\mu-R_f)}{2\sigma^3} \end{bmatrix} \begin{bmatrix} \sigma^2 & E(R_t-\mu)^3 \\ E(R_t-\mu)^3 & E(R_t-\mu)^4-\sigma^4 \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma} \\ \frac{-(\mu-R_f)}{2\sigma^3} \end{bmatrix} \\
&= \frac{\sigma^2}{\sigma^2} - 2\,\frac{\mu-R_f}{2\sigma^3}\,\frac{1}{\sigma}\,E(R_t-\mu)^3 + \frac{(\mu-R_f)^2}{4\sigma^6}\left(E(R_t-\mu)^4-\sigma^4\right)
\end{aligned}$$

[1] Or any other IID distributions with the same restrictions on the first four central moments as the normal.

[2] See his (A33) with the number of significant autocorrelations set to zero.

[3] Hence $E(R_t-\mu)^3 = 0$.

[4] Equivalent to $E(R_t-\mu)^4 = 3\sigma^4$.

$$
\begin{aligned}
\Leftrightarrow V_{IID} &= 1 - \mathrm{SR} \cdot \gamma_3 + \frac{1}{4} \cdot \mathrm{SR}^2 \left(\gamma_3 - 1 \right) \\
&= \underbrace{1 + \frac{1}{2} \cdot \mathrm{SR}^2}_{\text{(A4) of Lo (2002)}} - \mathrm{SR} \cdot \gamma_3 + \mathrm{SR}^2 \cdot \frac{\gamma_4 - 3}{4}
\end{aligned} \tag{2}
$$

where γ_3 and γ_4 measure skewness and kurtosis as defined by

$$
\begin{aligned}
\gamma_3 &= \frac{E(R_t - \mu)^3}{\sigma^3} \\
\gamma_4 &= \frac{E(R_t - \mu)^4}{\sigma^4}
\end{aligned}
$$

Skewness to the left ($\gamma_3 < 0$) and leptokurtosis ($\gamma_4 > 3$) increase the estimator's variance relative to the case of a normal distribution of returns. This can have profound impacts on the *quantitative* message of table 1 in Lo, which applies only to the case of normally distributed returns according to my argument. For instance, using the values for skewness and kurtosis of the Fama-French factor portfolios, the variances of table 1 in Lo (2002) are up to 70% off. In the ballpark of the Fama-French portfolios' Sharpe Ratios, the numbers are only 1–10% off. These results are reported in my tables 2, 3 and 4. The last column of each table reports the ratio of the standard errors computed under the assumption of normality with the standard errors computed in that table. If asymptotic standard errors are overstated by the normality assumption, values of this column are higher than 100%. My table 1 reproduces the values of Lo.

[Table 1 about here.]

[Table 2 about here.]

[Table 3 about here.]

[Table 4 about here.]

Keeping everything else fixed, true Sharpe Ratio's *magnitude* can change the verdict on whether standard errors are over- or understated under the normality assumption. This occurs when the signs of skewness and kurtosis have offsetting effects on equation (2) as is the case for HML and SMB (see tables 3 and 4).

Figure 1 shows results for estimated Sharpe Ratios from Monte Carlo simulations of returns using a log-normal distribution, hence a distribution skewed to the right. It shows a good fit between the CLT's normal distribution based on (2) and the histogram, whilst a CLT's distribution based on assuming normal returns would in this case tend to *overestimate* the standard errors. Similar plots are obtained from using a t-distribution or bootstraps of returns, for example the Fama-French portfolios. (Not reported here).

[Figure 1 about here.]

A final comment on using the Fama-French portfolios: There is nothing inherently particular to using these data for the present purpose. The main reason was that they came most handy for me in terms of data availability at the time of writing this up. As can be seen from the tables reported here, their Sharpe Ratios are relatively low compared to the other values of table 1 in Lo (2002). In their case, the corrected formula (1) matters less than for other assets having higher Sharpe Ratios and non-normal third and fourth moments.

2 GMM Setup

In order to estimate

$$\boldsymbol{\theta}_0 = \begin{bmatrix} \mu \\ \sigma^2 \end{bmatrix} = \begin{bmatrix} E\,R_t \\ E\left[(R_t - \mu)^2\right] \end{bmatrix}$$

we solve

$$\boldsymbol{g}_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{H}_t(\theta) = \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} R_t - \mu \\ (\mu - R_t)^2 - \sigma^2 \end{bmatrix} \stackrel{!}{=} \mathbf{0}$$

where $\boldsymbol{g}_T(\theta)$, $\boldsymbol{H}_t(\theta)$ are functions of $\boldsymbol{\theta}$ and the entire sample's data, respectively only date t data. So our estimators will be

$$\boldsymbol{\theta}_T = \begin{bmatrix} \hat{\mu} \\ \hat{\sigma}^2 \end{bmatrix} = \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} R_t \\ (\hat{\mu} - R_t)^2 \end{bmatrix}$$

and their joint limiting distribution is

$$\sqrt{T}(\boldsymbol{\theta}_T - \boldsymbol{\theta}_0) \xrightarrow{L} N\left(\mathbf{0}, \boldsymbol{V}_{\boldsymbol{\theta}}\right)$$

where

$$\begin{aligned} \boldsymbol{V}_{\boldsymbol{\theta}} &= \boldsymbol{D}_0^{-1}\boldsymbol{S}\boldsymbol{D}_0^{-1} \\ \boldsymbol{D}_0 &= \text{plim}\frac{\partial \boldsymbol{g}_T(\boldsymbol{\theta}_0)}{\partial \boldsymbol{\theta}} = \text{plim}\frac{\partial \boldsymbol{g}_T(\boldsymbol{\theta}_T)}{\partial \boldsymbol{\theta}} \\ &= \text{plim}\ \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} -1 & 0 \\ 2(\mu - R_t) & -1 \end{bmatrix} = -\boldsymbol{I} \end{aligned}$$

so that

$$\boldsymbol{V}_{\boldsymbol{\theta}} = \boldsymbol{S}$$

where $\boldsymbol{S}$ is the long-run variance-covariance matrix of $\boldsymbol{H}_t(\boldsymbol{\theta}_0)$

$$\begin{aligned}\boldsymbol{S} &= \sum_{\tau=-\infty}^{\tau=\infty} \boldsymbol{\Gamma}_\tau(\boldsymbol{H}_t(\boldsymbol{\theta}_0)) \\ &\overset{IID}{=} E(\boldsymbol{H}_t(\boldsymbol{\theta}_0)\boldsymbol{H}_t(\boldsymbol{\theta}_0)') \\ &= \begin{bmatrix} \sigma^2 & E(R_t-\mu)^3 \\ E(R_t-\mu)^3 & E(R_t-\mu)^4 - \sigma^4 \end{bmatrix}\end{aligned}$$

and $\boldsymbol{\Gamma}_\tau(\cdot)$ denotes τ-th order autocovariance matrix.

The Sharpe Ratio is a non-linear function $g(\cdot)$ of the vector $\boldsymbol{\theta}$. The variance of its limiting normal distribution can be computed by the delta method:

$$V_{\mathrm{SR}} = \frac{\partial g}{\partial \boldsymbol{\theta}'} \boldsymbol{V}_{\boldsymbol{\theta}} \frac{\partial g}{\partial \boldsymbol{\theta}'}'$$

where $\frac{\partial g}{\partial \boldsymbol{\theta}'}$ is the vector of partial derivatives of the Sharpe Ratio with respect to μ and σ^2:

$$\frac{\partial g}{\partial \boldsymbol{\theta}'}' = \begin{bmatrix} 1/\sigma \\ -(\mu - R_f)/(2\sigma^3) \end{bmatrix}$$

References

Goldberger, Arthur S. 1964. *Econometric Theory.* Wiley Publications in Statistics. New York, NY: John Wiley & Sons, Inc.

Greene, William H. 2000. *Econometric Analysis.* 4th edition. Upper Saddle River, NJ: Prentice Hall.

Lo, Andrew W. 2002. "The Statistics of Sharpe Ratios." *Financial Analysts Journal*, July/August, 36–52.

Theil, Henri. 1973. *Principles of Econometrics.* New York City, NY: John Wiley & Sons, Inc.

A Appendix: Matlab Code

Below you find some code I used to recalculate table 1 of Lo (2002) as well as for Monte Carlo simulations and bootstraps of Sharpe Ratios (under the maintained IID assumption). The m.-files can also be obtained from me directly, please mail to `emt@elmarmertens.org`.

A.1 Asymptotic Standard Errors

The following code was used to produce tables 1, 2, 3 and 4.

```
clear all
close all
clc

T  = [12 24 36 48 60 125 250 500];
SR = [0.5:0.25:3]';
g3 = 0;
g4 = 3;

load ffmonthly
datacomment = '(Simple monthly returns, 1926--2001)';
%load ffannual

ffdata = [rmrf, smb, hml];
desc = strvcat('RMRF', 'SMB', 'HML');

SRdata = round(mean(ffdata)' ./ std(ffdata,1)' * 100) / 100
SR  = [sort(SRdata); SR];
V      = 1 + 0.5 * SR.^2;
normalSEs = sqrt(V(:,ones(1,length(T))) ./ T(ones(length(V),1),:));
info.fmt = strvcat('%6.2f','%6.3f','%6.3f','%6.3f','%6.3f','%6.3f','%6.3f','%6.3f','%6.3f');

info.fid = fopen('lotab1normals.tab','wt');
lprint([SR normalSEs],info);
fclose(info.fid);
type lotab1normals.tab
info.fid = fopen('lotab1normals.tex','wt');
fprintf(info.fid,'Calculations based on values of $\\gamma_3=%4.1f$ and $\\gamma_4-3=%4.1f$ which match properties of %s.\n', g3, g4 - 3, 'normal distribution');
fclose(info.fid);
```

```
info.fmt = strvcat('%6.2f','%6.3f','%6.3f','%6.3f','%6.3f','%6.3f','%6.3f','%6.3f','%6.3f','%6.2f\%');
for i = 1 : 3

    data = ffdata(:,i);

    g3  = round(mean((data - mean(data)).^3) / std(data,1)^3)
    g4  = round(mean((data - mean(data)).^4) / std(data,1)^4)

    V       = 1 + 0.5 * SR.^2 - g3 * SR + (g4 - 3) / 4 * SR.^2;
    dataSEs = sqrt(V(:,ones(1,length(T))) ./ T(ones(length(V),1),:));

    filename = strcat('lotab1',desc(i,:),'.tab');
    info.fid = fopen(filename,'wt');
    %fprintf(info.fid,'\\midrule\n');
    %fprintf(info.fid,cat(2,'\\multicolumn{9}{c}{PANEL ', char(65 + i),' (', desc(i,:),'): $\\gamma_3=%4.1f$ and $\\gamma_4-3=%4.1f$}\\\\\n'), g3, g4 - 3);
    %fprintf(info.fid,'\\midrule\n');
    lprint([SR dataSEs normalSEs(:,1)./dataSEs(:,1)*100],info);
    fclose(info.fid);
    eval(cat(2,'type ',filename))
    info.fid = fopen(strcat('lotab1',desc(i,:),'.tex'),'wt');
    fprintf(info.fid,'Calculations based on values of $\\gamma_3=%4.1f$ and $\\gamma_4-3=%4.1f$ which match properties of %s portfolio %s. %s itself has a Sharpe Ratio of %4.2f.\n', g3, g4 - 3, desc(i,:), datacomment, desc(i,:), SRdata(i));
    fclose(info.fid);
end
```

A.2 Monte Carlo Simulations and Bootstraps

A.2.1 Log-Normal Distribution

The following code was used to produce figure 1. Thereafter I report code to produce similar output from simulating a t-distribution and bootstraps with the Fama-French factor portfolios.

```
clear all
close all
clc
```

```
N = 200;
T = 500;

% inputs
normalmu0    = 0.12;
normalsigma0 = .25;

%theoretical moments
alpha1 = lognmoment(1,normalmu0,normalsigma0);
alpha2 = lognmoment(2,normalmu0,normalsigma0);
alpha3 = lognmoment(3,normalmu0,normalsigma0);
alpha4 = lognmoment(4,normalmu0,normalsigma0);

mu0    = alpha1 - 1;
sigma0 = sqrt(alpha2 - alpha1^2);
m30    = alpha3 - 3 * alpha1 * alpha2 + 2 * alpha1^3;
m40    = alpha4 - 4 * alpha1 * alpha3 + 6 * alpha1^2 * alpha2 - 3 * alpha1^4;
g30    = m30 / sigma0^3;
g40    = m40 / sigma0^4;

SR0        = mu0 / sigma0
checkSR0   = (exp(normalmu0+normalsigma0^2/2) -1 )/ sqrt(exp(2*(normalmu0+normalsigma0^2))-exp(2*normalmu0+normalsigma0^2));
V0         = 1 + 0.5 * SR0.^2 - g30 .* SR0 + (g40 - 3) / 4 .* SR0.^2;
SE0T       = sqrt(V0 / T)
V0normal   = 1 + 0.5 * SR0.^2;
SE0Tnormal = sqrt(V0normal / T)

% simulation
R = exp(normrnd(normalmu0,normalsigma0,T,N));
mu    = mean(R) - 1;
sigma = std(R,1);
SR = mu ./ sigma;


dR = R - mu(ones(T,1),:);
m3 = mean(dR.^3);
m4 = mean(dR.^4);
g3 = m3 ./ sigma.^3;
g4 = m4 ./ sigma.^4;
```

```
mean(SR)
std(SR,1)
figure
%hbin = histfit2(SR,floor(N * 0.01),SR0,SE0T)
hbin = histfit3(SR,50,SR0,SE0T,SR0,SE0Tnormal);
```

A.2.2 t Distribution

This code produces output similar to figure 1 based on a t-distribution (results not reported here).

```
clear all
close all
clc

N = 1000;
T = 250;

% inputs
mu0    = .15;
scale  = 0.3;
v      = 100;

%theoretical moments
sigma0 = scale * sqrt(tmoment(2,v));
m30    = 0;
m40    = scale^4 * tmoment(4,v);
g30    = m30 / sigma0^3;
g40    = m40 / sigma0^4;
SR0        = mu0 / sigma0
V0         = 1 + 0.5 * SR0.^2 - g30 .* SR0 + (g40 - 3) / 4 .* SR0
    .^2;
SE0T       = sqrt(V0 / T)
V0normal   = 1 + 0.5 * SR0.^2;
SE0Tnormal = sqrt(V0normal / T)

% simulation
R = scale * trnd(v,T,N) + mu0;
mu    = mean(R);
sigma = std(R,1);
SR = mu ./ sigma;
```

```
% some sample stats
dR = R - mu(ones(T,1),:);
m3 = mean(dR.^3);
m4 = mean(dR.^4);
g3 = m3 ./ sigma.^3;
g4 = m4 ./ sigma.^4;

% output
mean(SR)
std(SR,1)
figure
% Note: Below I fit the CLT's distribution around the *sample
    mean*
hbin = histfit3(SR,50,SR0,SE0T,mean(SR),SE0Tnormal);
```

A.2.3 Bootstraps

This code produces output similar to figure 1 based on bootstraps from the Fama-French returns (results not reported here).

```
clear all
close all
clc

% do bootstraps

N = 10000
withdata = 0
load ffannual rmrf
%load ffmonthly rmrf

R0 = rmrf;
clear rmrf;

T = length(R0);
mu0    = mean(R0);

sigma0 = std(R0,1);
g30    = skewness(R0);
g40    = kurtosis(R0);;

SR0        = mu0 / sigma0
```

```
V0         = 1 + 0.5 * SR0.^2 - g30 .* SR0 + (g40 - 3) / 4 .* SR0.^2;
SE0T       = sqrt(V0 / T)
V0normal   = 1 + 0.5 * SR0.^2;
SE0Tnormal = sqrt(V0normal / T)

if withdata
    [SR, ind] = bootstrp(N,'sharperatio', R0 ,0);
    R     = R0(ind);
    mu    = mean(R);
    sigma = std(R,1);
    g3 = skewness(R);
    g4 = kurtosis(R);
    disp([g30 mean(g3)])
    disp([g40 mean(g4)])
else
    SR = bootstrp(N,'sharperatio', R0 ,0);
end

mean(SR)
std(SR,1)
figure
% "CLT" plotted around empirical mean
hbin = histfit3(SR,50,SR0,SE0T,mean(SR),SE0Tnormal);
```

Figure 1: SR's sampling distribution from log-normal (N=10,000)

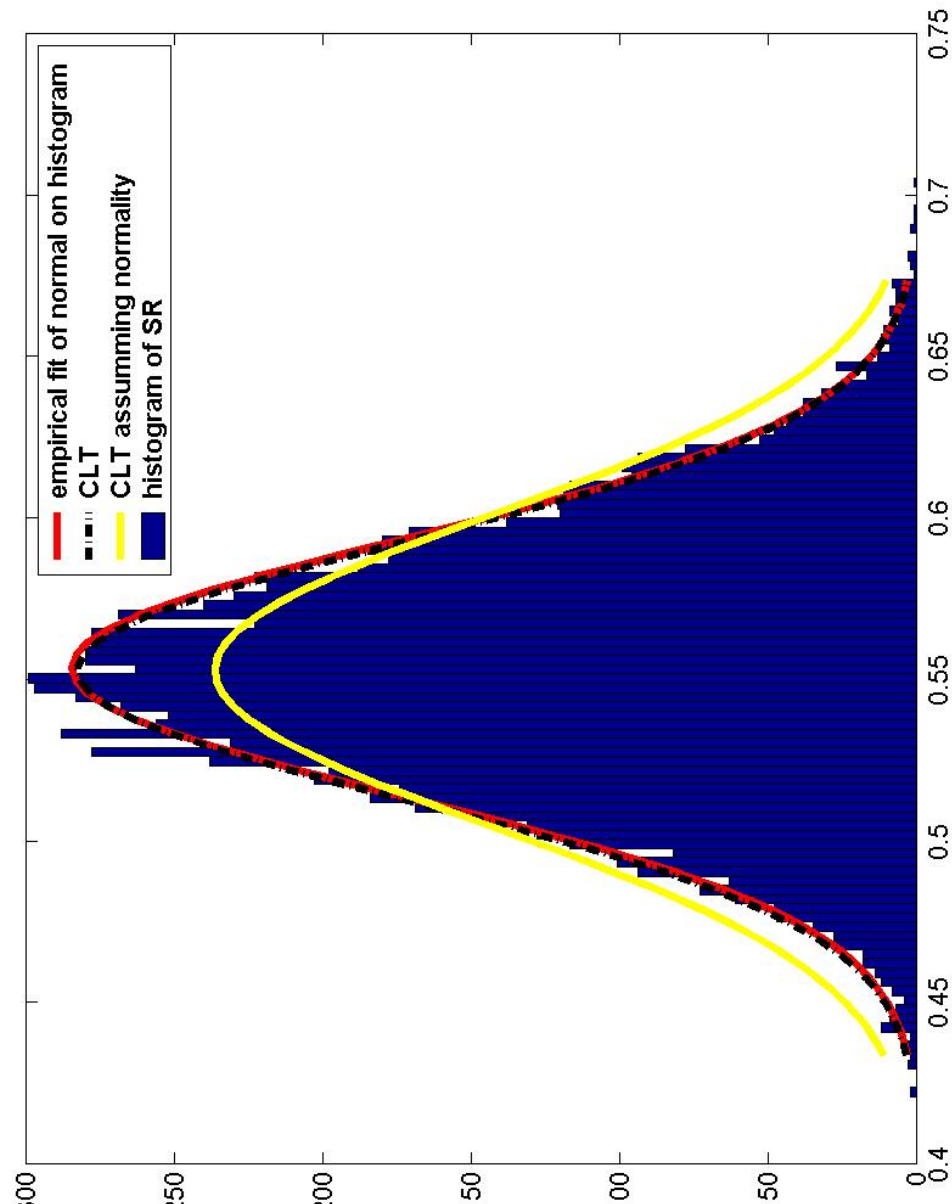

Table 1: Asymptotic standard errors of estimated Sharpe Ratios (IID) with normal moments

	T							
SR	12	24	36	48	60	125	250	500
0.06	0.289	0.204	0.167	0.144	0.129	0.090	0.063	0.045
0.11	0.290	0.205	0.167	0.145	0.129	0.090	0.063	0.045
0.12	0.290	0.205	0.167	0.145	0.130	0.090	0.063	0.045
0.50	0.306	0.217	0.177	0.153	0.137	0.095	0.067	0.047
0.75	0.327	0.231	0.189	0.163	0.146	0.101	0.072	0.051
1.00	0.354	0.250	0.204	0.177	0.158	0.110	0.077	0.055
1.25	0.385	0.272	0.222	0.193	0.172	0.119	0.084	0.060
1.50	0.421	0.298	0.243	0.210	0.188	0.130	0.092	0.065
1.75	0.459	0.325	0.265	0.230	0.205	0.142	0.101	0.071
2.00	0.500	0.354	0.289	0.250	0.224	0.155	0.110	0.077
2.25	0.542	0.384	0.313	0.271	0.243	0.168	0.119	0.084
2.50	0.586	0.415	0.339	0.293	0.262	0.182	0.128	0.091
2.75	0.631	0.446	0.364	0.316	0.282	0.196	0.138	0.098
3.00	0.677	0.479	0.391	0.339	0.303	0.210	0.148	0.105

Calculations based on values of $\gamma_3 = 0.0$ and $\gamma_4 - 3 = 0.0$ which match properties of normal distribution.

Table 2: Asymptotic Standard Errors of estimated Sharpe Ratios (IID) with moments of RMRF portfolio.

	T								
SR	12	24	36	48	60	125	250	500	**% dev.**
0.06	0.290	0.205	0.167	0.145	0.130	0.090	0.064	0.045	99.64
0.11	0.293	0.207	0.169	0.147	0.131	0.091	0.064	0.045	98.82
0.12	0.294	0.208	0.170	0.147	0.131	0.091	0.064	0.046	98.60
0.50	0.368	0.260	0.212	0.184	0.165	0.114	0.081	0.057	83.21
0.75	0.448	0.317	0.259	0.224	0.200	0.139	0.098	0.069	72.97
1.00	0.540	0.382	0.312	0.270	0.242	0.167	0.118	0.084	65.47
1.25	0.639	0.452	0.369	0.320	0.286	0.198	0.140	0.099	60.25
1.50	0.743	0.525	0.429	0.372	0.332	0.230	0.163	0.115	56.64
1.75	0.849	0.601	0.490	0.425	0.380	0.263	0.186	0.132	54.08
2.00	0.957	0.677	0.553	0.479	0.428	0.297	0.210	0.148	52.22
2.25	1.067	0.754	0.616	0.533	0.477	0.331	0.234	0.165	50.85
2.50	1.177	0.832	0.680	0.589	0.526	0.365	0.258	0.182	49.81
2.75	1.288	0.911	0.744	0.644	0.576	0.399	0.282	0.200	49.01
3.00	1.399	0.990	0.808	0.700	0.626	0.434	0.307	0.217	48.38

Calculations based on values of $\gamma_3 = 0.0$ and $\gamma_4 - 3 = 8.0$ which match properties of RMRF portfolio (Simple monthly returns, 1926–2001). RMRF itself has a Sharpe Ratio of 0.12. Last column shows percentage of the asmyptotic standard errors of Table 1 relative to this table's values. (This ratio changes only with the Sharpe Ratio's value not the number of observations, T.). If asymptotic standard errors are overstated by the normality assumption, values of this column are higher than 100%.

Table 3: Asymptotic Standard Errors of estimated Sharpe Ratios (IID) with moments of HML portfolio.

	T								
SR	12	24	36	48	60	125	250	500	**% dev.**
0.06	0.273	0.193	0.158	0.137	0.122	0.085	0.060	0.042	105.78
0.11	0.263	0.186	0.152	0.132	0.118	0.082	0.058	0.041	110.00
0.12	0.262	0.185	0.151	0.131	0.117	0.081	0.057	0.041	110.75
0.50	0.298	0.210	0.172	0.149	0.133	0.092	0.065	0.046	102.90
0.75	0.397	0.281	0.229	0.198	0.178	0.123	0.087	0.061	82.32
1.00	0.520	0.368	0.300	0.260	0.233	0.161	0.114	0.081	67.94
1.25	0.655	0.463	0.378	0.327	0.293	0.203	0.143	0.101	58.86
1.50	0.794	0.561	0.458	0.397	0.355	0.246	0.174	0.123	53.01
1.75	0.936	0.662	0.540	0.468	0.419	0.290	0.205	0.145	49.06
2.00	1.080	0.764	0.624	0.540	0.483	0.335	0.237	0.167	46.29
2.25	1.225	0.866	0.707	0.613	0.548	0.380	0.268	0.190	44.27
2.50	1.371	0.970	0.792	0.686	0.613	0.425	0.300	0.212	42.76
2.75	1.518	1.073	0.876	0.759	0.679	0.470	0.333	0.235	41.59
3.00	1.665	1.177	0.961	0.832	0.744	0.516	0.365	0.258	40.67

Calculations based on values of $\gamma_3 = 2.0$ and $\gamma_4 - 3 = 15.0$ which match properties of HML portfolio (Simple monthly returns, 1926–2001). HML itself has a Sharpe Ratio of 0.11. Last column shows percentage of the asmyptotic standard errors of Table 1 relative to this table's values. (This ratio changes only with the Sharpe Ratio's value not the number of observations, T.). If asymptotic standard errors are overstated by the normality assumption, values of this column are higher than 100%.

Table 4: Asymptotic Standard Errors of estimated Sharpe Ratios (IID) with moments of SMB portfolio.

	T								
SR	12	24	36	48	60	125	250	500	**% dev.**
0.06	0.274	0.194	0.158	0.137	0.123	0.085	0.060	0.042	105.46
0.11	0.266	0.188	0.154	0.133	0.119	0.082	0.058	0.041	108.82
0.12	0.265	0.187	0.153	0.133	0.119	0.082	0.058	0.041	109.32
0.50	0.346	0.245	0.200	0.173	0.155	0.107	0.076	0.054	88.47
0.75	0.477	0.338	0.276	0.239	0.213	0.148	0.105	0.074	68.45
1.00	0.629	0.445	0.363	0.315	0.281	0.195	0.138	0.097	56.20
1.25	0.790	0.558	0.456	0.395	0.353	0.245	0.173	0.122	48.78
1.50	0.955	0.675	0.551	0.477	0.427	0.296	0.209	0.148	44.08
1.75	1.122	0.793	0.648	0.561	0.502	0.348	0.246	0.174	40.93
2.00	1.291	0.913	0.745	0.645	0.577	0.400	0.283	0.200	38.73
2.25	1.461	1.033	0.843	0.730	0.653	0.453	0.320	0.226	37.13
2.50	1.631	1.154	0.942	0.816	0.730	0.505	0.357	0.253	35.94
2.75	1.802	1.274	1.041	0.901	0.806	0.558	0.395	0.279	35.02
3.00	1.974	1.396	1.140	0.987	0.883	0.612	0.432	0.306	34.30

Calculations based on values of $\gamma_3 = 2.0$ and $\gamma_4 - 3 = 21.0$ which match properties of SMB portfolio (Simple monthly returns, 1926–2001). SMB itself has a Sharpe Ratio of 0.06. Last column shows percentage of the asmyptotic standard errors of Table 1 relative to this table's values. (This ratio changes only with the Sharpe Ratio's value not the number of observations, T.). If asymptotic standard errors are overstated by the normality assumption, values of this column are higher than 100%.