Guess the Number:
A Game on Rational Equilibria and Bubbles in Financial Markets

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January 2002

In this game, you need to guess a number which is lower than that of your average competitor

RULES OF THE GAME

Pick a number between 0 and 100 ($0 \leq x \leq 100$)

The winning number will be determined as being as close as possible to two-thirds of the average number chosen

A possible outcome could be an equilibrium of everybody's guesses

EQULIBRIUM SOLUTION

Iterative Reasoning:

The maximum average possible is 100, 2/3 of 100 ≈ 67 . . .

But if everybody picks 67, I better say 2/3 • 67 ≈ 45 . . .

But if everybody picks 45, I say 2/3 • 45 = 30 . . .

But if everybody picks 30, I say 2/3 • 30 = 20 . . .

and so on . . . and so on . . .

In the end, we get only a stable solution, when **everybody picks 0** [*)]

Definition of Equilibrium:

- No player wants to change his guess, once he knows the result of the game

- We get a stable solution in repeated plays

- Key is that everybody is rational, knows that everybody else is rational, knows that everybody else knows that everybody is rational and so forth . . .

Should we expect such an equilibrium as the actual outcome for our game?

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In a competition of the FT, the winner estimated how deep his competitors looked at the problem

OUTCOME OF NEWSPAPER COMPETITION
From the Financial Times "Mastering Finance" Competition, 1997

- 1,486 FT readers entered a mailing contest in 1997
- **Average Guess: 18.91**. Two-thirds: $12.6 \approx 13$
- Some players from Oxford tried to „move the market" by picking 100. They changed the winning number from 12 to 13!
- Other plays yield similar results:
  - With MBA students numbers between 13 - 20 typically win
  - Economists tend to pick numbers close to zero (Guess why!)

The winner reasoned:

„**70 per cent of readers compute: viable range 0-66.**
2/3 of mid-point = 22.

30 per cent of readers reason: If I bid x, to win:
$0.3 \cdot x + 0.7 \cdot 22 = 3/2x$.
Solution x=13."

The reasonings of contestants in the FT competition varied

TYPICAL REASONINGS

From the Financial Times "Mastering Finance" Competition, 1997

„So behaviourists observe a bod
An FT reader, ergo clever sod
He knows the competition and will fight em
So reduces the number ad infinitum."

„The answer should be naught (0)
. . . but Labour won."

Zero as equilibrium solution

Pick 1, not zero, as a „hedge" against irrationality

Finding the „average guy"

Anticipate how far the others think!

„My dad knows an average amount about numbers and markets and he bottled out at 10."

„Over 67 only interests fools;
So over 45 implies innummeracy rules.
1 to 45 random averages 23.
So logic indicates 15, leaving 10 to me."

Source: Mertens; Richard Thaler's comments on the FT competition (FT, 16 June 1997)

With some simple formulas, we can express our first "models" of what happens in the game

FORMULAS FOR SIMPLE INTERPRETATIONS

Depth of reasoning

- Based on the iterative solution we can back out the steps "n" taken by the participants in a game with result "x":
$n = \ln(x/100) / \ln(2/3)$

- With a starting value of 50 we get
$n = \ln(x/50) / \ln(2/3)$

"Winning Formula" of FT contest

- Winning number "x" as function of the share "w" of smart players:
$x(w) = 22 \cdot (w-1) / (w-3/2)$

- The inverse, w(x), determines the share of smart players *):
$w(x) = (3/2 \cdot x - 22) / (x - 22)$

Market Power

- How many additional votes of "100" would be necessary to change the results by "1", given that "N" people have voted on average "x"?
$m(N, x) = N / (99 - x)$

- For N<100, a single vote of 100 changes the result!

Fooling the smarts by market moving

- "N-S" bid 22, "S-m" plug this into the FT-formula while "m-1" move the market and "1" tries to win with "x":
$x(N,m,S) = 200 \cdot (m-1) + 44 \cdot (N-S)$
$\cdot [1 + (S-m) / (3/2 \cdot N - S)]$

- Market moving is easy, but winning is not obvious!

Source: Mertens; *) Sensible values only for $x < 22 \cdot 2/3 \approx 15$

Like in our game, economic problems involve the anticipation of human behavior

GUESS-THE-NUMBER AND FINANCE

- Standard models suppose that markets will tend to be at rational equilibria, hence the search for equilibrium prices in finance

- The game shows, that it is not sufficient if everybody is rational. Instead, everybody else has to know about it and everybody has to know that everybody knows and so forth . . .

- Otherwise, it could be rational to suppose that an „irrational" result occurs. For example, investments in „bubbles" like the new economy (if it was really a bubble) might be rational, *given* we could expect prices to rise forever. (Of course, prices cannot grow without limits, that is why economists impose „super-rationality" in order to rule out such bubbles)

Economics in general and Finance in particular is not about mechanical solutions, it **is about anticipation of human behavior**. And behavior might change because of our anticipations

. . . and also because of financial models

Source: Mertens

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KEYNES ON VALUATION
General Theory (1936), p.152ff.

Prices anticipate the anticipated

„[P]rofessional investment may be likened to those newspaper competitions in which the competitors have to pick out the . . . prettiest faces . . .

[T]he prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole . . .

[In the end, we] have reached the third degree, where we devote our intelligences to anticipating what average opinion expects the average opinion to be"

Prices efficiently reflect fundamentals

„[T]he existing market valuation, however arrived at, is uniquely *correct* in relation to our existing knowledge of the facts which will influence the yield of the investment, and that it will only change in proportion to changes in this knowledge . . .

[T]he only risk [the investor] runs is that of genuine change in the news *over the near future*"

(Please note that Keynes is usually not regarded as being an advocate of efficient market theory!)

Do you think the prettiest girl, will win in this beauty contest?

Source: Mertens

There are two broad schools of financial valuation

TWO MODES OF FINANCIAL VALUATION

Anticipation of tomorrow's price

- An asset is worth, what the next buyer is willing to pay for it
- Fundamental value is nice, but our cash-flow from the asset is tomorrow's price
- There is always a greater fool, willing to buy the asset. We only need to get in early

$$P_0 = \frac{CF_1}{1+R} + \frac{P_1}{1+R}$$

„Castle in the Air"

Intrinsic Value

- An asset is worth the discounted value of its cash-flows (DCF)
- There may be short-run fluctuations, but only the DCF is sustainable
- In "efficient" markets, only fundamental news change prices by surprise

$$P_0 = \frac{CF_1}{1+R} + \frac{CF_2}{(1+R)^2} + \frac{P_2}{(1+R)^2}$$

$$= \sum_{t=1}^{\infty} \frac{CF_t}{(1+R)^t} \qquad \frac{P_1}{1+R}$$

„ Firm Foundation"