FIXED INCOME PRODUCTS & DERIVATIVES

STUDY CENTER
GERZENSEE

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AGENDA

- Key Concept: No-Arbitrage
- Fixed Income Products
- Interest Rate Risk Managment
- Options

The focus of the next two lectures is on relative pricing

ASSET PRICING

Absolute Pricing

- How is asset price linked to fundamental, macroeconomic sources of risk?
- E.g.: level of stock market vis-à-vis economic growth
- CAPM (in theory), consumption based models

„No Free Lunch "
Return commensurate with Risk

Relative Pricing

- Given some asset prices, what can we say about some other securities? (a.k.a derivatives)
- E.g.: Stocks of Palm v 3Com
- Derivative pricing, CAPM (in practice)
- Larry Summers: "Ketchup economics"

„No Happy Meal"
Derivative price is replication cost

AGENDA

- Key Concept: No-Arbitrage
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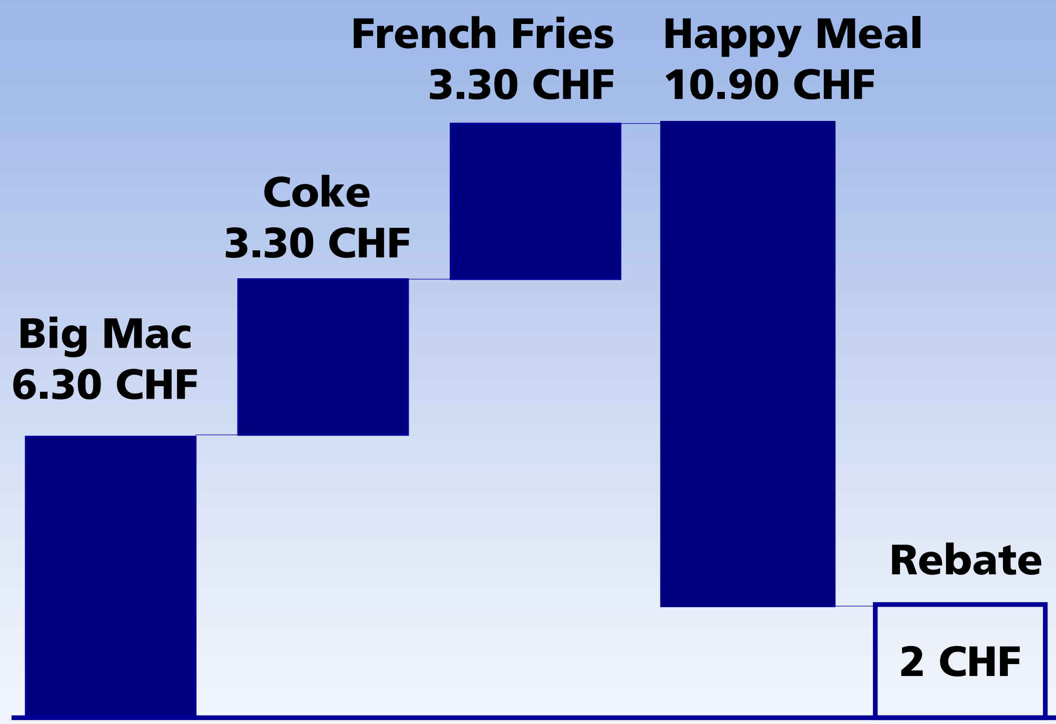
- Options

A Happy Meal is cheaper than the sum of its components

HAPPY MEAL PRICING



To be answered next:

- **Shouldn't everybody buy happy meals and sell the pieces?**
- **Why is there no arbitrage at McDonalds?**
- **Why should there be arbitrage in financial markets?**

Here is the definition of an arbitrage opportunity ...

ARBITRAGE OPPORTUNITY

Criteria

Costs nothing	No investment outlays Zero price portfolio, usually involving short (borrowing) and long (lending) positions
Never makes a loss	No future capital requirements Losses in one leg of portfolio must always be offset by gains on other leg
Some Chance of Profit	At least in some circumstances, net profits accrue. (Net profits can vary across states)

No arbitrage (NA) is a necessary, but not sufficient condition for equilibrium

NO ARBITRAGE

No Equilibrium with Arbitrage Opportunities

Perfect Markets:

- No trading costs
- No counterparty risk
- Everybody can buy (long) or sell (short) any asset in any quantity

More better than less:

- Everybody would want to buy into arbitrage opportunity, nobody should want to sell
- Unlimited demand at zero supply

Pricing is only relative:

- For example, 4% borrowing and 5% lending:
- No clue whether borrowing too low, lending too high or both

Arbitrage can occur, but only in limited scale for limited time: *Off* equilibrium!

... and here are some examples

EXAMPLES OF ARBITRAGE

	Costless	No Loss	Profit: maybe	Profit: sure	**Arbitrage**
Lottery Ticket					
- for free	✓	✓	✓	✗	✓
- for 1 cent	✗	✓	✓	✗	✗
Borrowing at 2% and lending at 4%	✓	✓	-	✓	✓
Borrowing JPY at 1% and lending USD at 4%	✓	✗	✓	✗	✗

Apart from terminology financial portfolios are much like meals

FOOD RETAIL V WHOLESALE FINANCE

BACKUP

McDonalds		Finance
Coke, Burger, Fries	⟷	Primitive assets (often indirect: Coke, Coke & Burger, Coke & Burger & Fries)
Prices of Coke, Burger Fries	⟷	State prices
A meal	⟷	Complex security / portfolio
Happy Meal	⟷	Arbitrage opportunity, since meal should cost same as sum of components
You can only buy from McDonalds, no resale	⟷	Investors can buy and sell whatever quantity they want

AGENDA

- Key Concept: No-Arbitrage
- **Fixed Income Products**
- Interest Rate Risk Managment
- Options

We will look at the following products

OVERVIEW FI PRODUCTS

Product	Trade-Off
Zero Coupon Bond	Today v a later date
Coupon Bond	Today v several later dates
Forward Rate	Tomorrow v after tomorrow
Floating Rate Bond	Today v tomorrow
Interest Rate Swap	Long-term asset v short-term liability

Some „jargon“is handy when talking about fixed income

TALK THE TALK

GLOSSARY

- **Bond** Debtor's Promise to repay borrowed money at fixed dates
- **Principal** Amount due (a.k.a. par value, notional). Here always equal to one
- **Maturity** Time until last payment
- **Duration** Average time for all payments (see later)
- **Coupon** Intermediate payment, usually fixed percentage of notional
- **Zero bond** Bond without coupon, only principal due (also: bullet)
- **Yield** Interest rate (see later: Yield-to-Maturity)

Take the term structure of interest rates as given

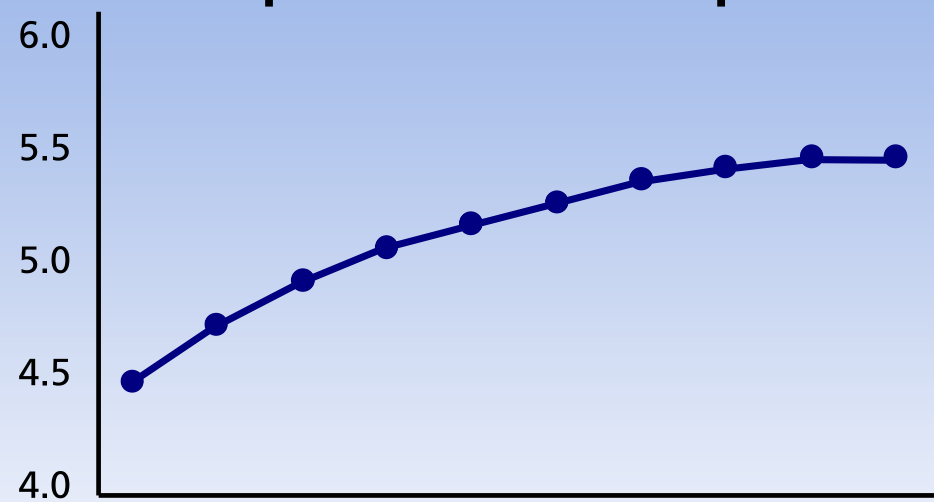
FIXED INCOME SETTING

Consider:

- Fixed amounts due at fixed dates
- No default
- „When, not whether, will payments occur?“

Basic prices: Interest rates between now and future dates

Spot Rate Curve in % p.a.



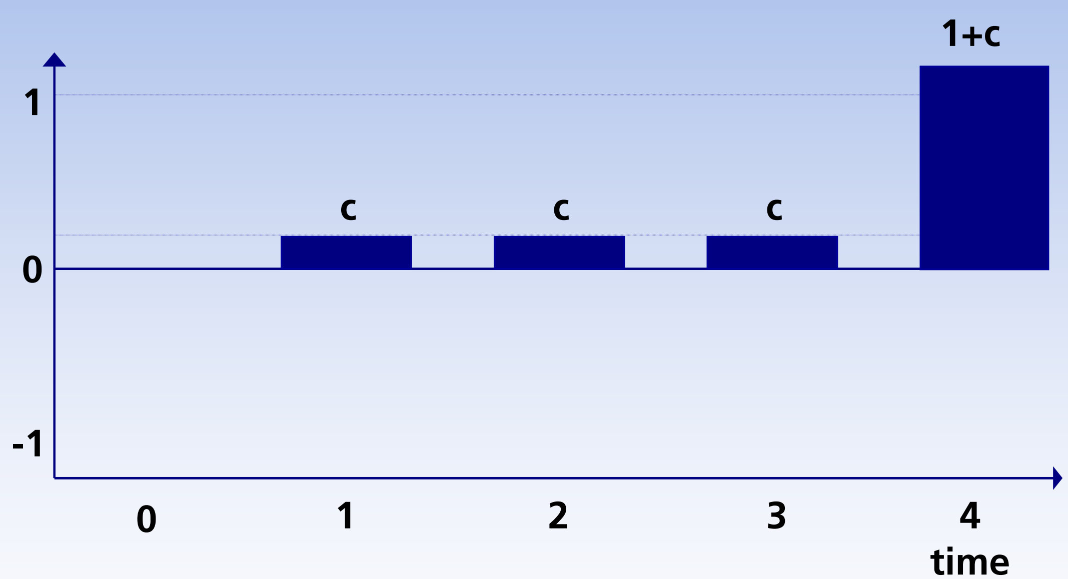
	1	2	3	4	5	6	7	8	9	10
i(0,t)	4.5	4.75	4.95	5.1	5.2	5.3	5.4	5.45	5.5	5.5

A coupon bond holder receives multiple payments.
We focus first on the individual payments ...

COUPON BOND

A coupon bond has regular payments prior to maturity

Payoffs



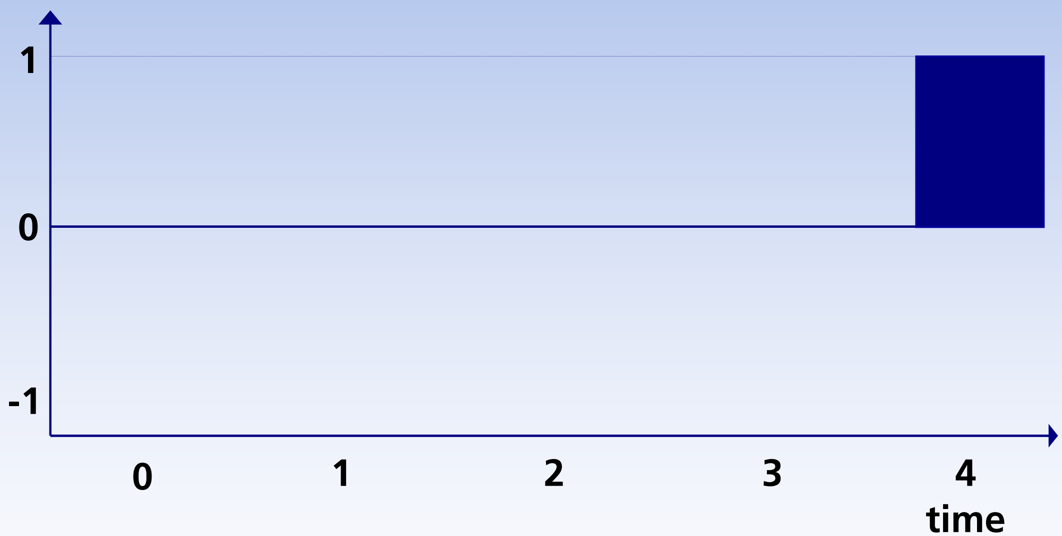
Zero bonds are the basic building blocks of fixed income pricing

ZERO BOND

TERMSHEET

A bond without coupons
Single payment at maturity: principal (here equal unity)

Payoffs



Pricing

$$B(0,T) = \frac{1}{[1+i(0,T)]^T}$$

Example

$B(0,1) = 1.045^{-1} = 0.96$

$B(0,2) = 1.0475^{-2} = 0.91$

$B(0,3) = 0.87$

$B(0,4) = 0.82$

Notes

Yield a.k.a. „spot rate", „zero rate"

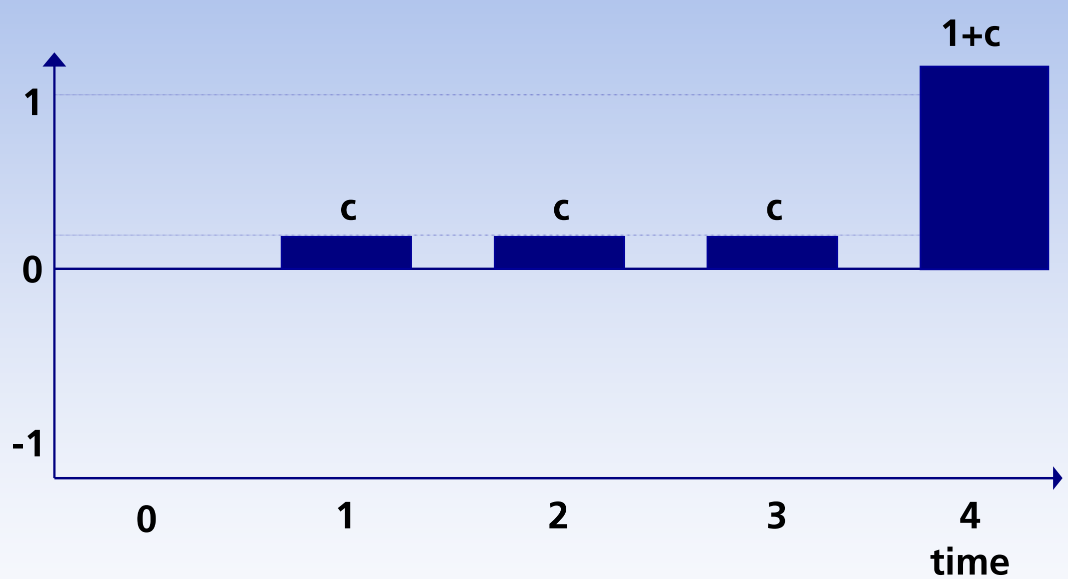
A coupon bond is just a collection of zero bond payments

COUPON BOND

TERMSHEET

A coupon bond has regular payments prior to maturity

Payoffs



Pricing

$$B(0,T,c) = \sum_{t=1}^{T} \frac{c}{[1+i(0,t)]^t} + \frac{1}{[1+i(0,T)]^T}$$

Examples

$$B(0,4,10\%) = 2.3$$

$$B(0,4,0.5\%) = 1.02$$

Par bonds have a coupon equal to their yield-to-maturity

COUPON BOND AT PAR

For which coupon, would the bond trade at par?

$$c = \frac{1-[1+i(0,T)]^{-T}}{\sum_{t=1}^{T}[1+i(0,t)]^{-t}} = i$$

Par Value (=1)

$$(1+c)\cdot[1+i(0,T)]^{-T}$$
$$+\,c\cdot[1+i(0,T-1)]^{-(T-1)}$$
$$+\quad\vdots$$
$$+\,c\cdot[1+i(0,1)]^{-1}$$

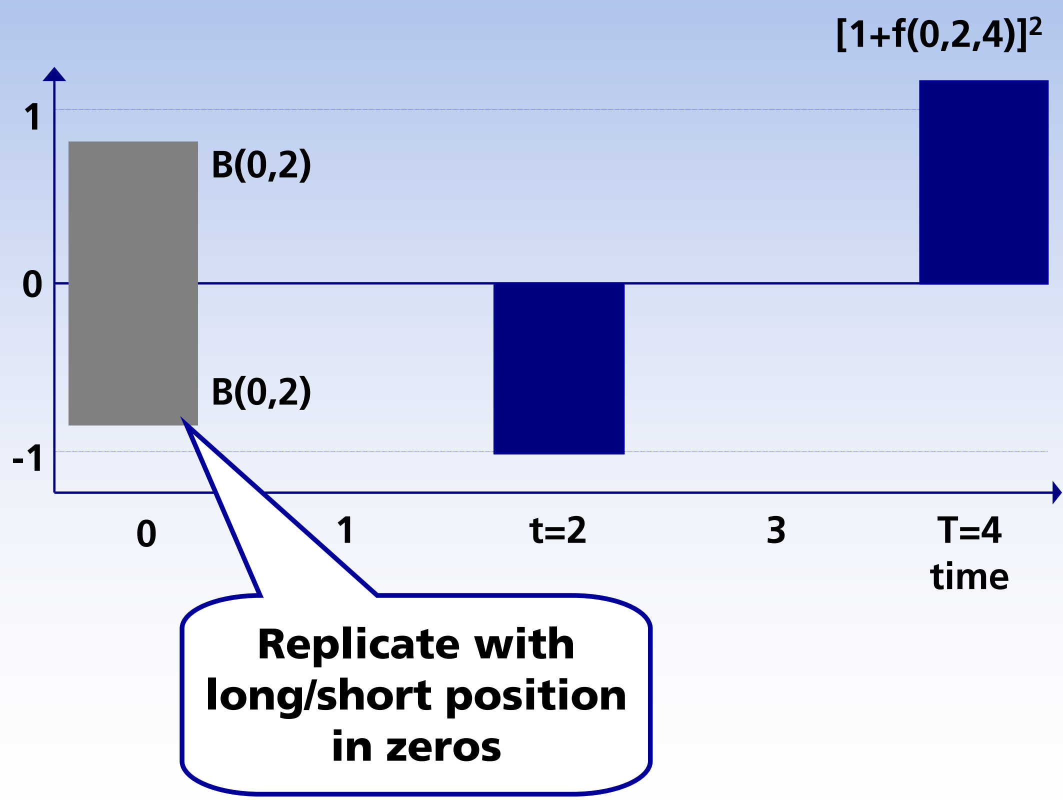
= Bond price

FORWARD RATES

TERMSHEET

An agreement today, to borrow money between t and T at rate f(0,t,T)

Payoffs



Pricing

$$B(0,t) = B(0,T) \cdot [1+f(0,t,T)]^{T-t}$$

$$f(0,t,T) = \frac{[1+i(0,T)]^{T/T-t}}{[1+i(0,t)]^{t/T-t}} - 1$$

Examples

$$f(0,2,4) = \left(\frac{1.051^4}{1.0475^2} \right)^{1/2}$$

$$= 8.06\%$$

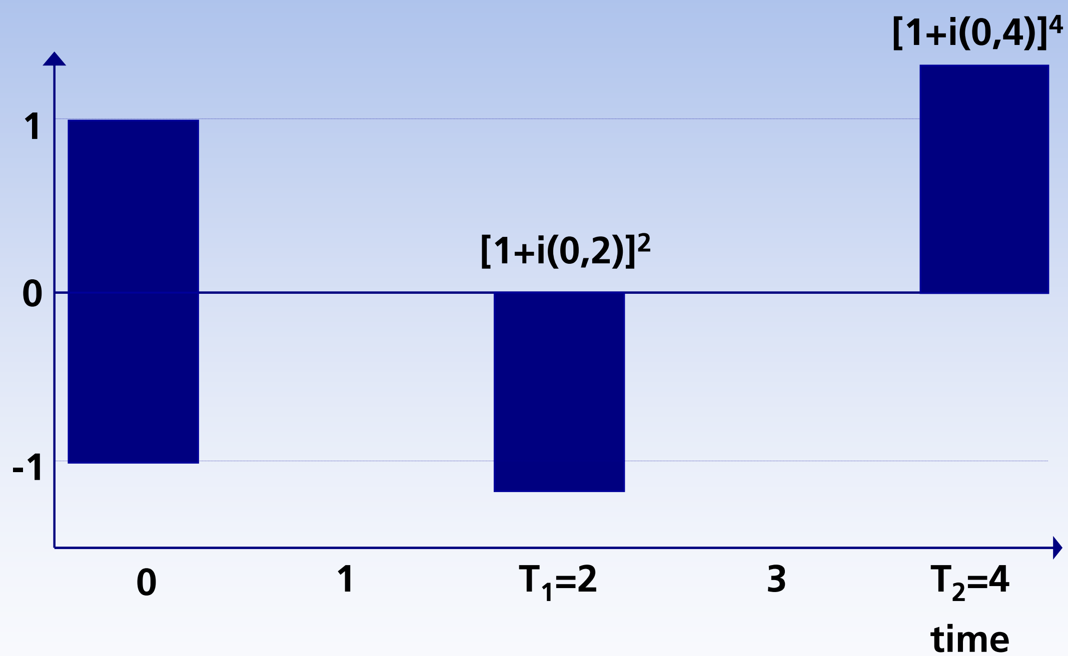
The carry trade bets on differences between future spot rates and today's forward rates

CARRY TRADE

BACKUP

Example of a simple, zero-investment portfolio.
Borrow short-term and lend long-term with different maturities

Payoffs



Notes

- No initial capital, zero price
- Effectively a forward contract
- Risk of reinvestment/ repricing at T_1
- Profitable if

$$B(1,T_2-T_1)[1+i(0,T_2)]^{T_2} > [1+i(0,T_1)]^{T_1}$$

- Or, if future spot rate below today's forward

$$i(1,T_2-T_1) < f(0,T_1,T_2)$$

Multi-period zeros are like a sequence of forward contracts

ZEROS AND FORWARDS

BACKUP

No-Arbitrage implies that zero rates are geometric averages of forward rates

$$1 + i(0,T) = \prod_{t=1}^{T}[1 + f(0,t-1,t)]^{1/T}$$

Likewise, forward rates are geometric averages of forward rates

$$1 + f(0,t,t+n) = \prod_{j=0}^{n-1}[1 + f(0,t+j,t+j+1)]^{1/n}$$

The math is simple:

$$B(0,T) = B(0,1)\frac{B(0,2)}{B(0,1)}\frac{B(0,3)}{B(0,2)}\cdots\frac{B(0,T)}{B(0,T-1)}$$

What do forward rates tell us about market expectations of future spot rates?

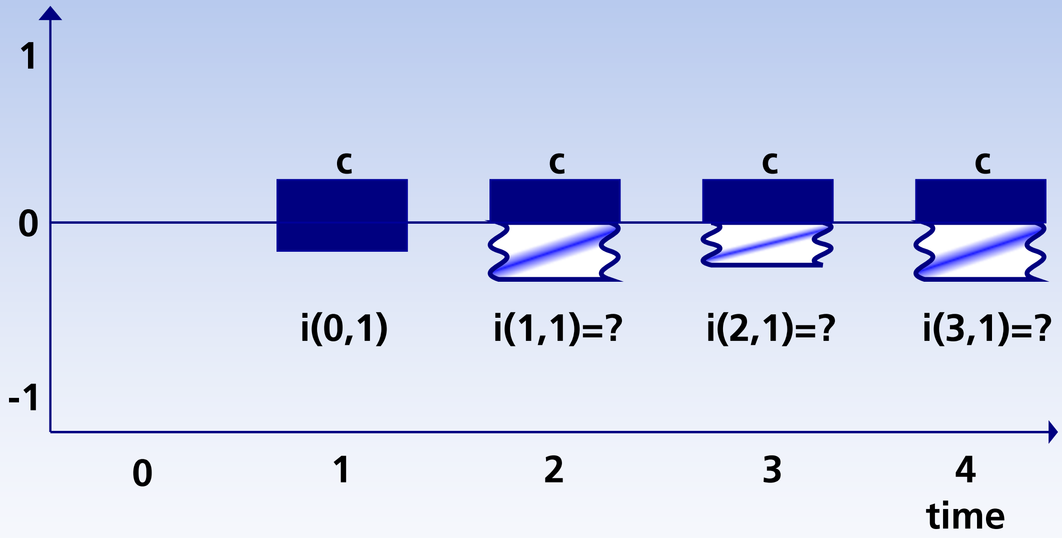
Do they say more than current spot rates?

Swaps offset fixed versus variable payments. To understand them better we will look at "floating rate bonds" first

INTEREST RATE SWAP

Exchange of interest rate payments: fixed v floating. No exchange of principal
Interest rates calculated for „notional" principal

Payoffs



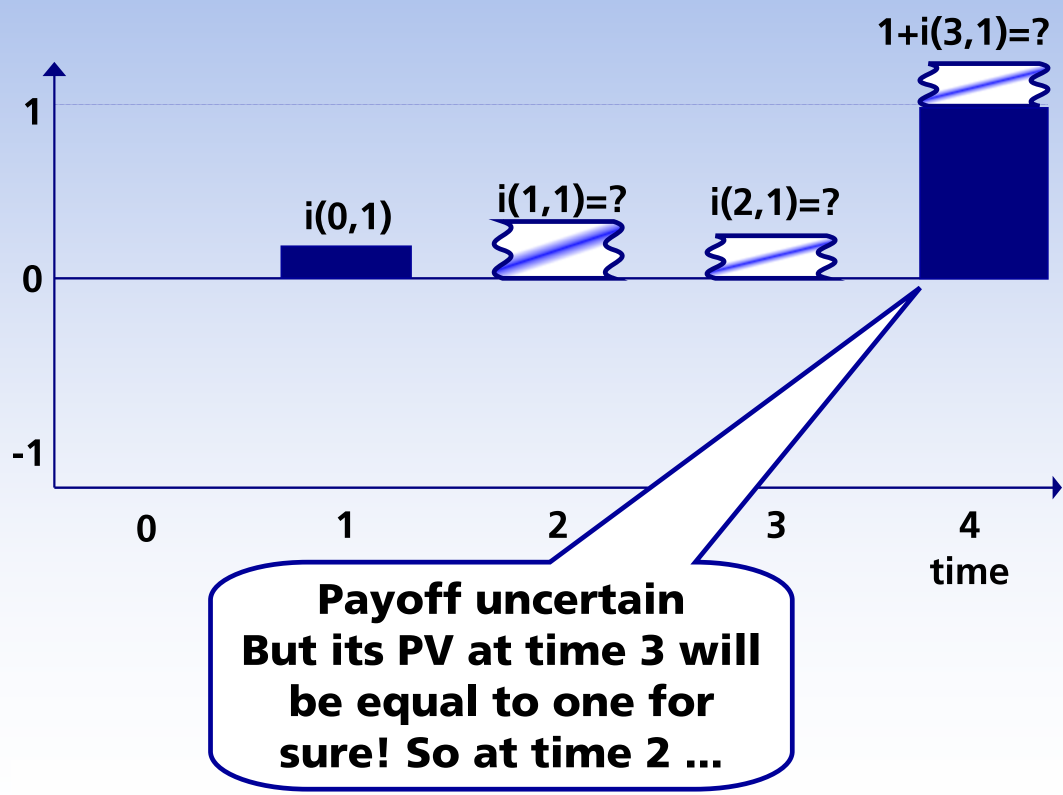
Payments of a floater are determined one period in advance, but unknown today

FLOATING RATE BOND

TERMSHEET

Variable coupons.
Next coupon always set equal to then prevailing, one-period spot rate

Payoffs



Pricing

$$B(0,T,float) = 1$$

$$D(0,T,float) = 1$$

Notes

- Effectively, a one-period zero bond
- Implied "option" to reinvest at future spot rates worthless under our wholesale conditions

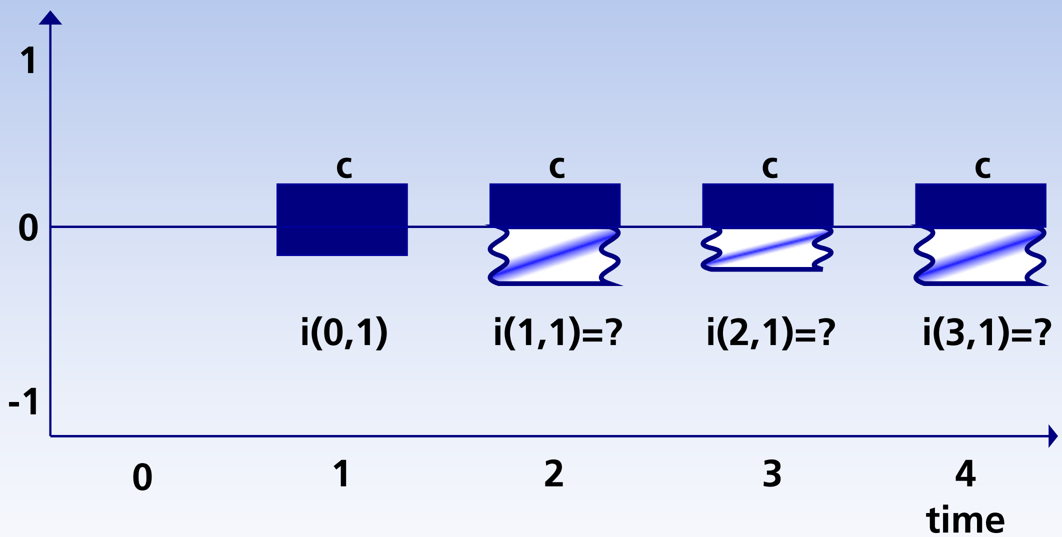
Swaps match a floater with a fixed coupon bond

INTEREST RATE SWAP

TERMSHEET

Exchange of interest rate payments: fixed v floating. No exchange of principal
Interest rates calculated for „notional“ principal

Payoffs



Pricing

- A portfolio of coupon bond (long) and floater (short) with same principal
- Here: “fixed receiver“, opposite picture for fixed payer
- Coupon such that initial value is zero: **Fixed-leg is par bond!**

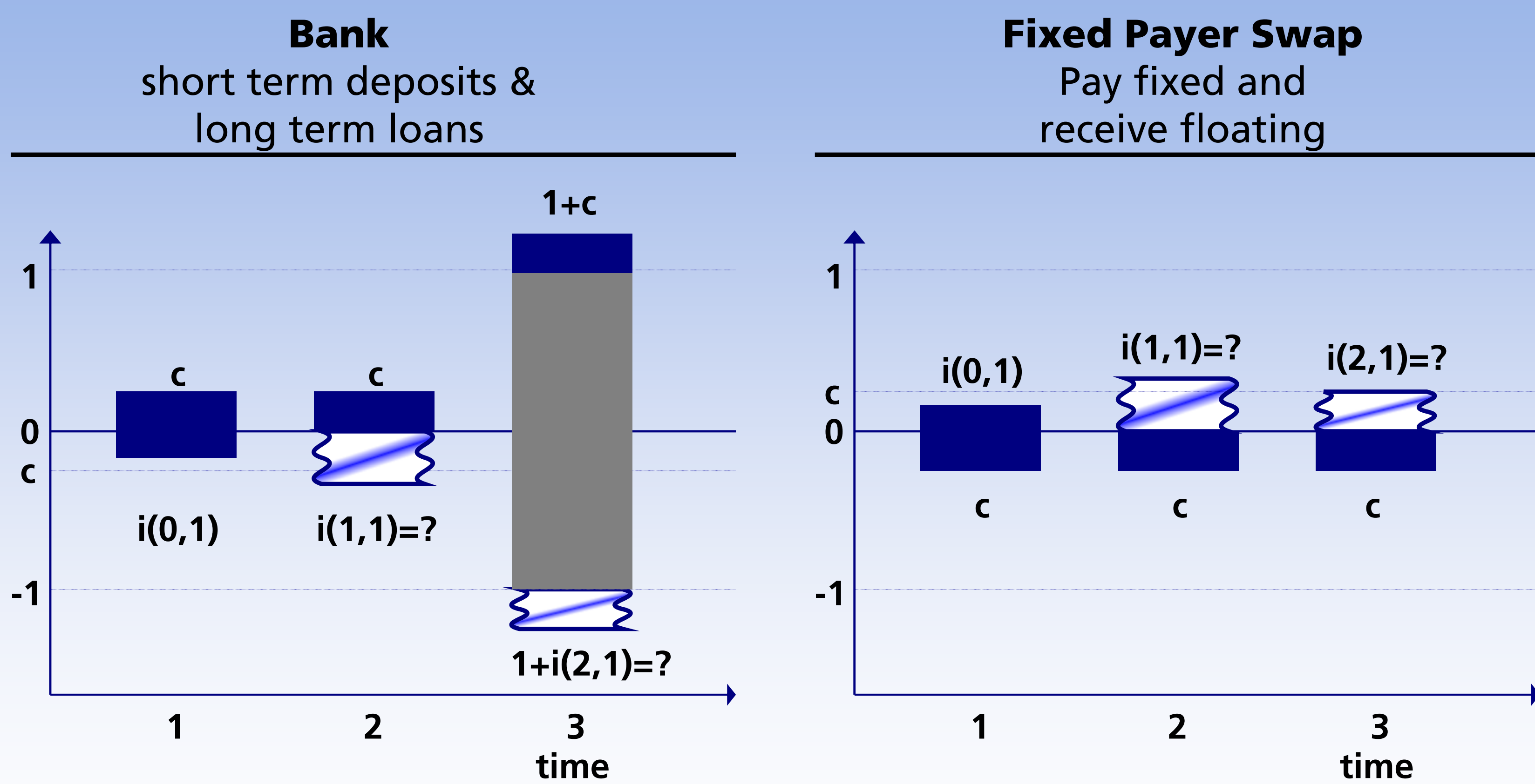
$$c = \frac{1-[1+i(0,T)]^{-T}}{\sum_{t=1}^{T}[1+i(0,t)]^{-t}}$$

Example

4y year swap-rate: 5.08%

Swaps can simultaneously match the asset & liability profile of bank's maturity risks. And against *any* move in the term structure!

ALM WITH SWAPS



When you get lost in applications, try following this little system

COMPASS TO NA PRICING

Basic Assets

- Collect Prices
- Study Payoffs

Replicate the derivative with a portfolio of assets

Derivative Price

- Equals replication cost
- Asset prices times asset weights in replication

If you cannot replicate: Absolute pricing (or more assets)

AGENDA

- Key Concept: No-Arbitrage
- Fixed Income Products
- Interest Rate Risk Managment
- Options

A bond's Internal Rate of Return (IRR) is also known as Yield-to-Maturity

YIELD-TO-MATURITY

If the TS were flat and remained flat, what would be the return to bondholder if she reinvested all coupons until maturity?

$$i = \sqrt[T]{\frac{\sum_{t=0}^{T} c(1+i)^{T-t} + 1}{B(0,T,c)}} - 1$$

$$B(0,T,c) = \sum_{t=1}^{T} \frac{c}{(1+i)^t} + \frac{1}{(1+i)^T}$$

**If the TS were flat.
At which level would it match the bond price?**

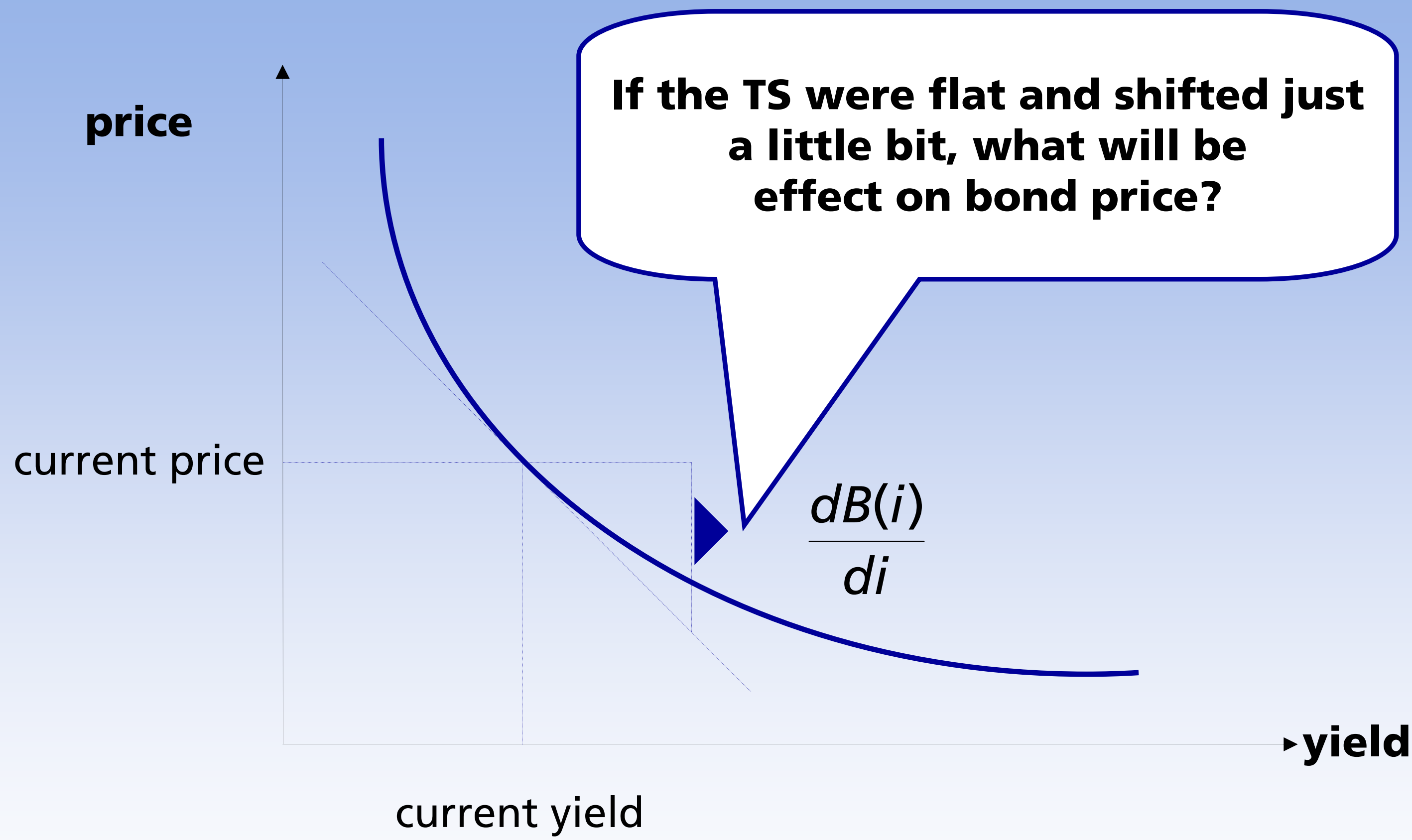
Caveats

Like with any IRR ...

- TS usually not flat
- Yield-to-maturity is complex average of actual spot rates
- TS not constant, reinvestment risk is substantial
- Sensitive to size of coupon
- Specific to each bond!

For small yield changes we can look at linear approximations to price changes

PRICE YIELD RELATIONSHIP

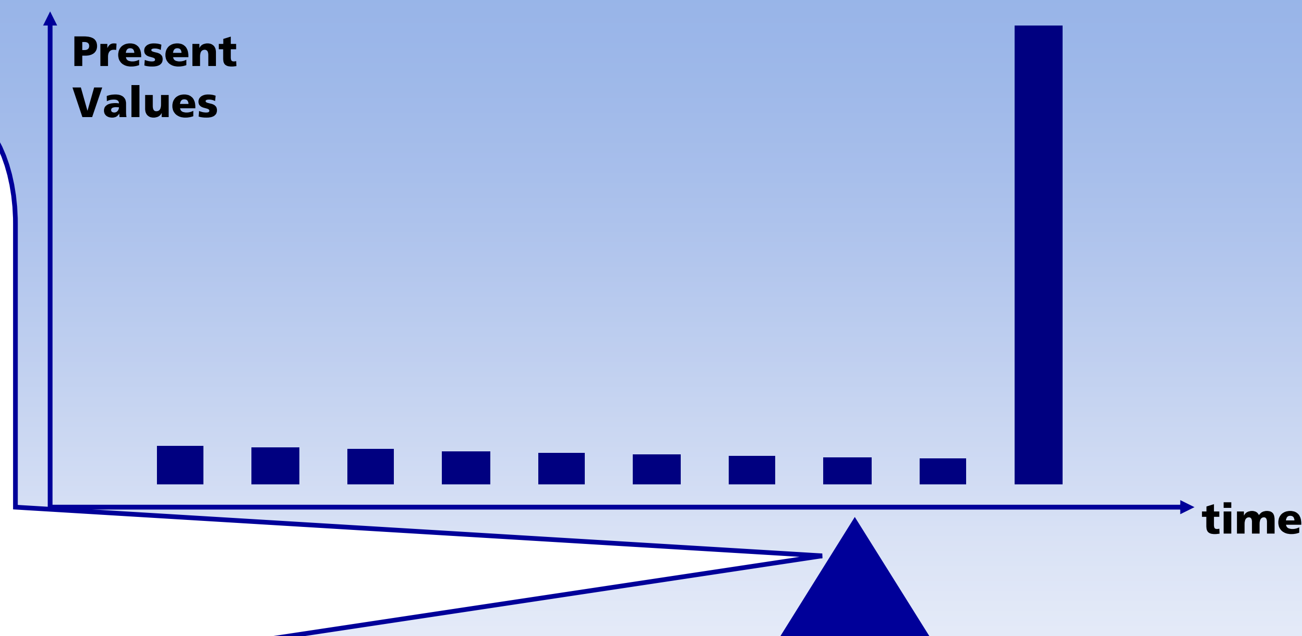


**To understand price sensitivity, duration is important.
It measures the center of gravity of the payoffs' present values**

DURATION

What is the average time until all claims are paid?

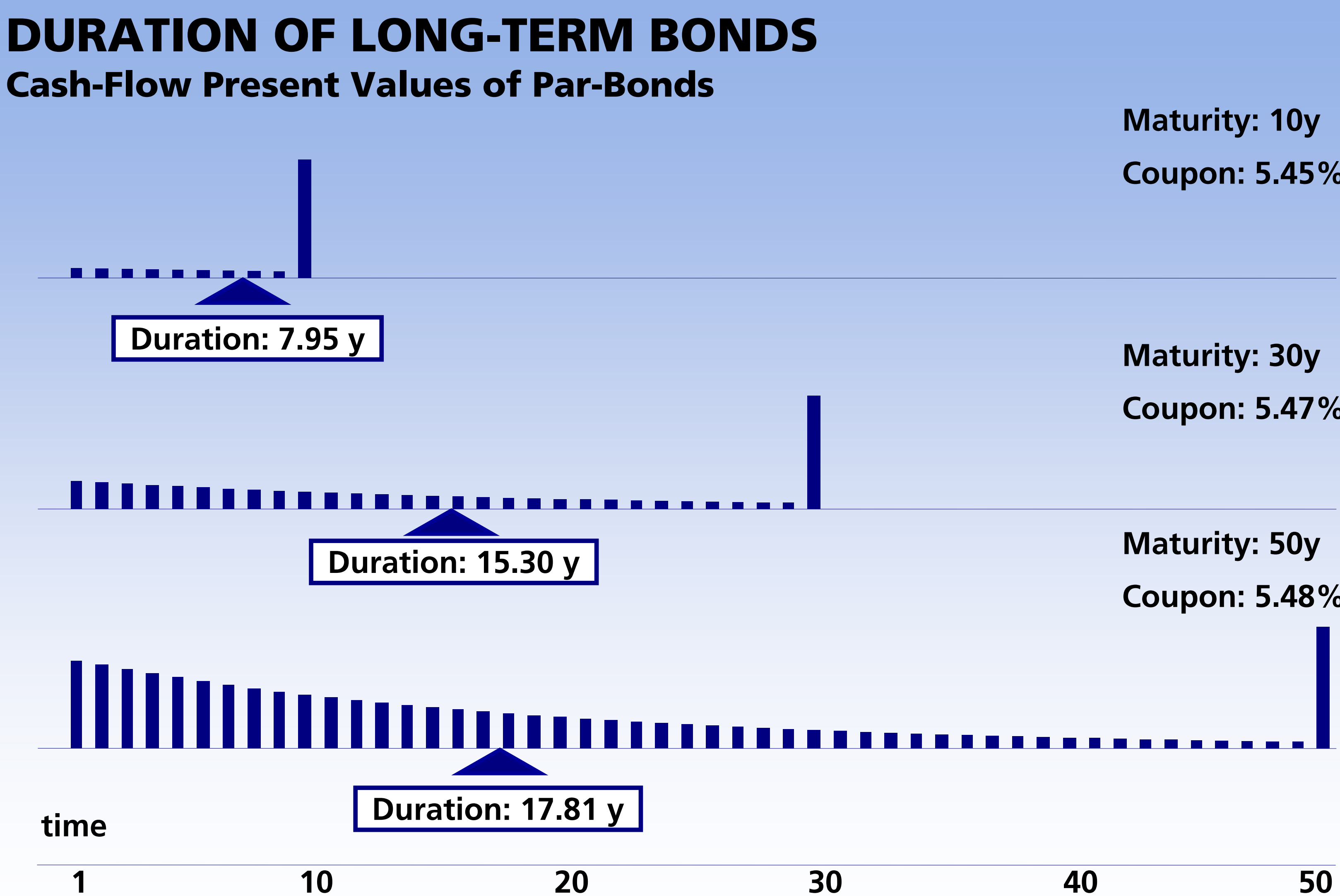
(... and let us average with present-value weights)



$$D(0,T,c) = \sum_{t=1}^{T} t \, \frac{c(1+i)^{-t}}{B(0,T,c)} + T \, \frac{(1+i)^{-T}}{B(0,T,c)}$$

$$D_{eff}(0,T,c) = \sum_{t=1}^{T} t \, \frac{c[1+i(0,t)]^{-t}}{B(0,T,c)} + T \, \frac{[1+i(0,T)]^{-T}}{B(0,T,c)}$$

Compared to long term bonds, ultra-long-term bonds do not add much duration



Valuation using term structure example, spot rates beyond 10y equal 5.5%

That is why duration is so useful: It is proportional to approximate price changes when yield changes

PRICE SENSITIVITY AND DURATION

… The effect of yield change on bond price is proportional to duration!

$$\frac{dB(i)}{di} = -\frac{B(i)}{1+i} D$$

$$B(i) = \sum_{t=1}^{T} \frac{c}{(1+i)^t} + \frac{1}{(1+i)^T}$$

$$D = \left(\sum_{t=1}^{T} t \frac{c}{(1+i)^t} + T \frac{1}{(1+i)^T} \right) B(i)^{-1}$$

Note

- Higher duration, higher yield risk
- Only for small changes in yield-to-maturity and parallel shifts in flat TS
- D/(1+i) a.k.a. "modified Duration"
- Extensions
  - Effective Duration (uses actual spot rates)
  - Key Rate Duration (moves of single spot rate)

Portfolio duration should match the duration of your liabilities

IMMUNIZATION

Problem

- How to protect bond portfolio's value in H periods, F_H, against changes in yield-to-maturity?
- Again: parallel shifts of flat TS

Solution:

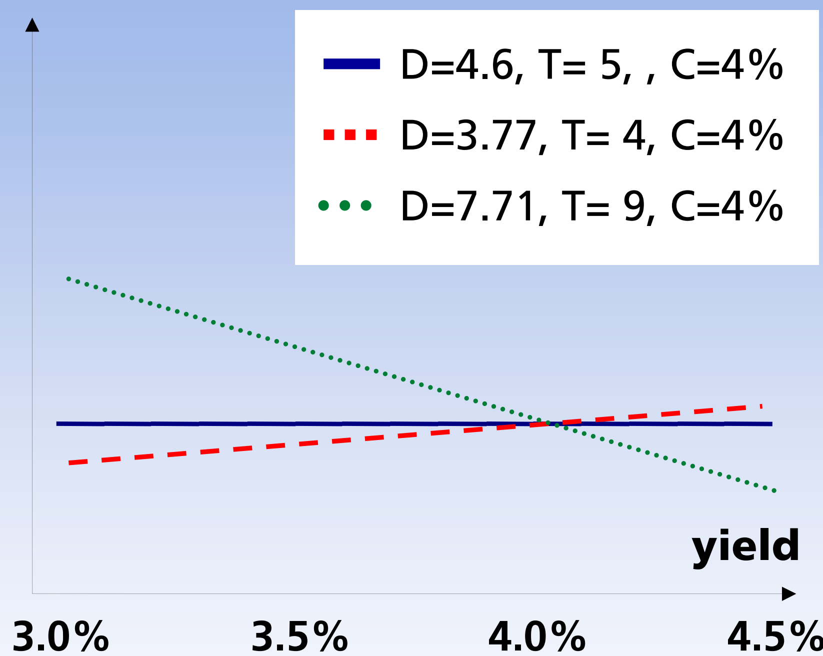
- Construct portfolio such that duration equals horizon H

$$F_H(i) = B(i)(1+i)^H$$

$$\frac{dF_H(i)}{di} = H(1+i)^{H-1}B(i) + \frac{dB(i)}{di}(1+i)^H$$

$$\frac{dF_H(i)}{di} = 0 \quad \text{for} \quad H = D$$

Future Values in 4.6 years



To improve upon the linear approximation, look at convexity

CONVEXITY

BACKUP

Note

- Convexity is "good" since: lower price fall if yield increases, higher gain if yield falls
- Convexity greater if payoffs more dispersed over time

$$C = \frac{d^2B(i)}{di^2}\frac{1}{B(i)}$$

$$\frac{\Delta B(i)}{B(i)} \approx \frac{1}{B(i)}\left(\frac{dB(i)}{di}di + \frac{1}{2}\frac{d^2B(i)}{di^2}(di)^2\right) = \left(\frac{-D}{(1+i)}di + \frac{1}{2}C(di)^2\right)$$

See de La Grandville chapters 7 & 8 for more

It is straightforward to extend the previous tools to bond portfolios

BOND PORTFOLIOS

BACKUP

General Principle:
„Price everything as a bundle of zero bonds“

- Zero bond prices B(t), B(0) = 1
- Payoffs: CF(t) (t=0 … T)
- payoff's value-weight: $w(t) = \frac{CF(t)B(t)}{P}$

Portfolio Price and Duration:

$$P = \sum_{t=0}^{T} CF(t)B(t)$$

$$D = \sum_{t=0}^{T} t \frac{CF(t)B(t)}{P}$$

$$= \sum_{t=0}^{T} t \, w(t)$$

Applied to Bond Portfolio:

- Portfolio with bonds i = 1… N
- Bonds price P_i, quantities q_i, Durations D_i and payoffs $CF_i(t)$
- value-weights $w_i = \frac{q_i P_i}{P}$
- Cash Flows $CF(t) = \sum_{i=1}^{N} q_i CF_i(t)$

Portfolio Price and Duration:

$$P = \sum_{t=0}^{T} \sum_{i=1}^{N} q_i CF_i(t) B(t)$$

$$= \sum_{t=0}^{T} CF(t)B(t) = \sum_{i=1}^{N} w_i P$$

$$D = \sum_{i=1}^{N} w_i D_i$$

Duration: only for non-zero price and when yield-to-maturities not different

Always remember the assumptions behind simple analysis with yield / duration / convexity

CAVEAT

The previous section on immunization, duration and convexity looked at changes in yield-to-maturity

This is best understood for parallel shifts with flat term structures

When you aggregate in a portfolio, all bonds should thus have the same yield

These techniques are common and within the above limits useful. But be careful when looking at bonds with different yield-to-maturity

To think about differences in yields and non-parallel shifts, look at *effective duration* and *key rate durations*

YOU HAVE BEEN WARNED ;-)

AGENDA

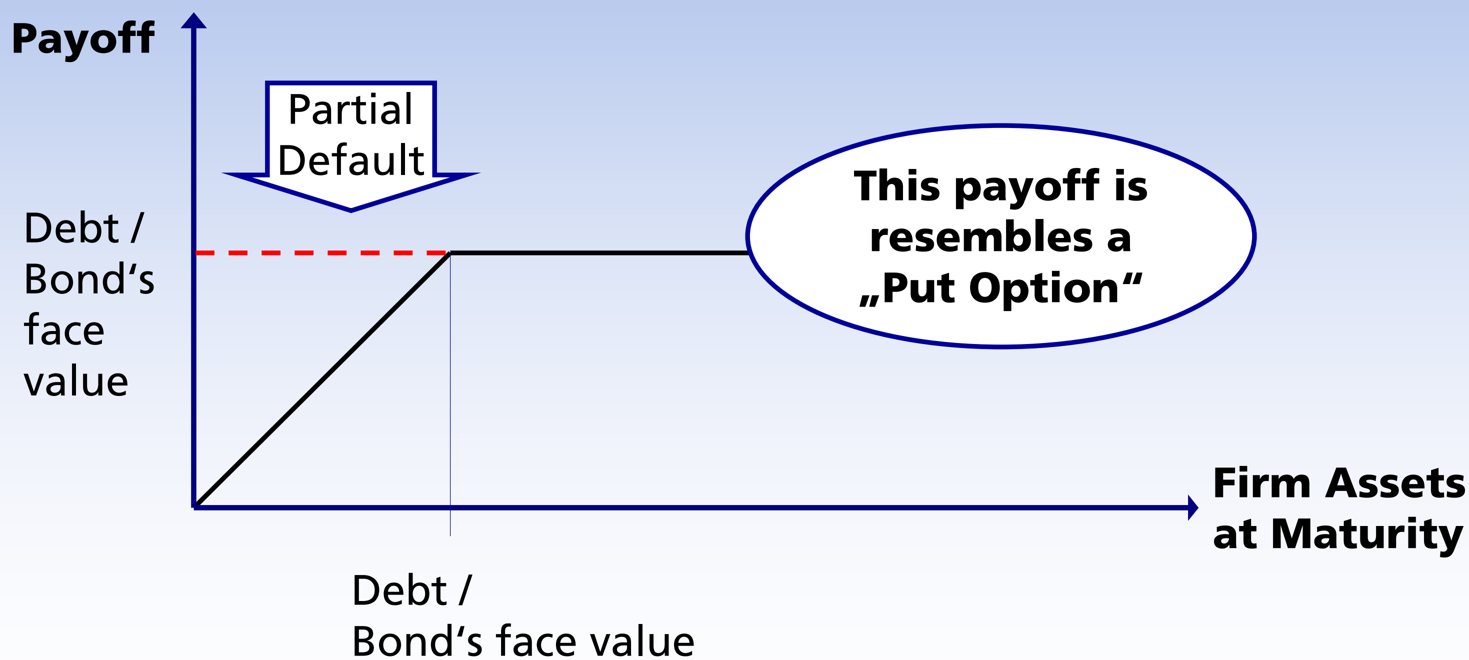
- Key Concept: No-Arbitrage
- Fixed Income Products
- A First Glance at Risk Managment
- Options

Once we want to consider the default risk of corporate bonds, we are in the world of options, that is our next topic

CORPORATE BOND

Zero bond issue from a risky business with limited liability is not default free

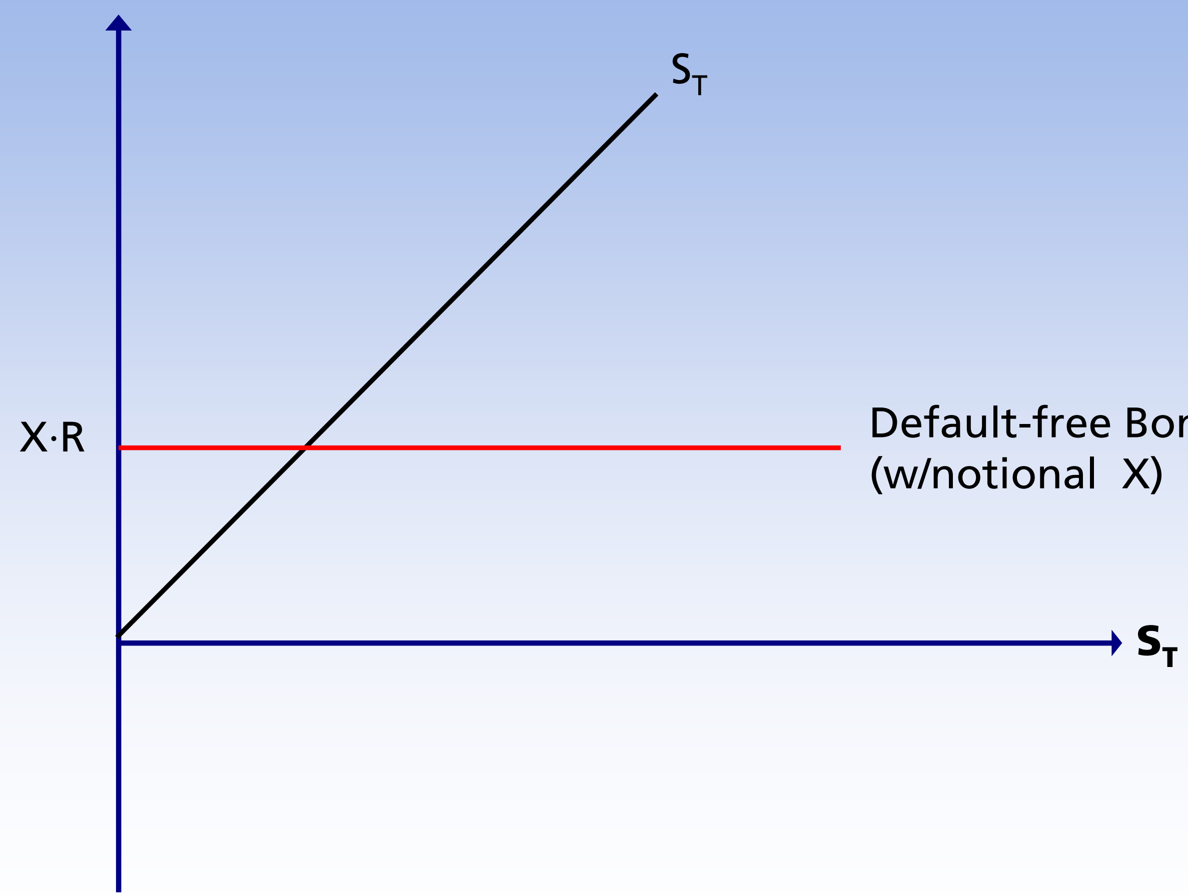
Need to think about issuer‘s ability to repay, in other words: need to look at firm‘s asset value at bond‘s maturity



Now we look at a single point in time, „T“, but across different states (the various prices of the underlying stock S_T)

STOCK AND BOND

Payoffs

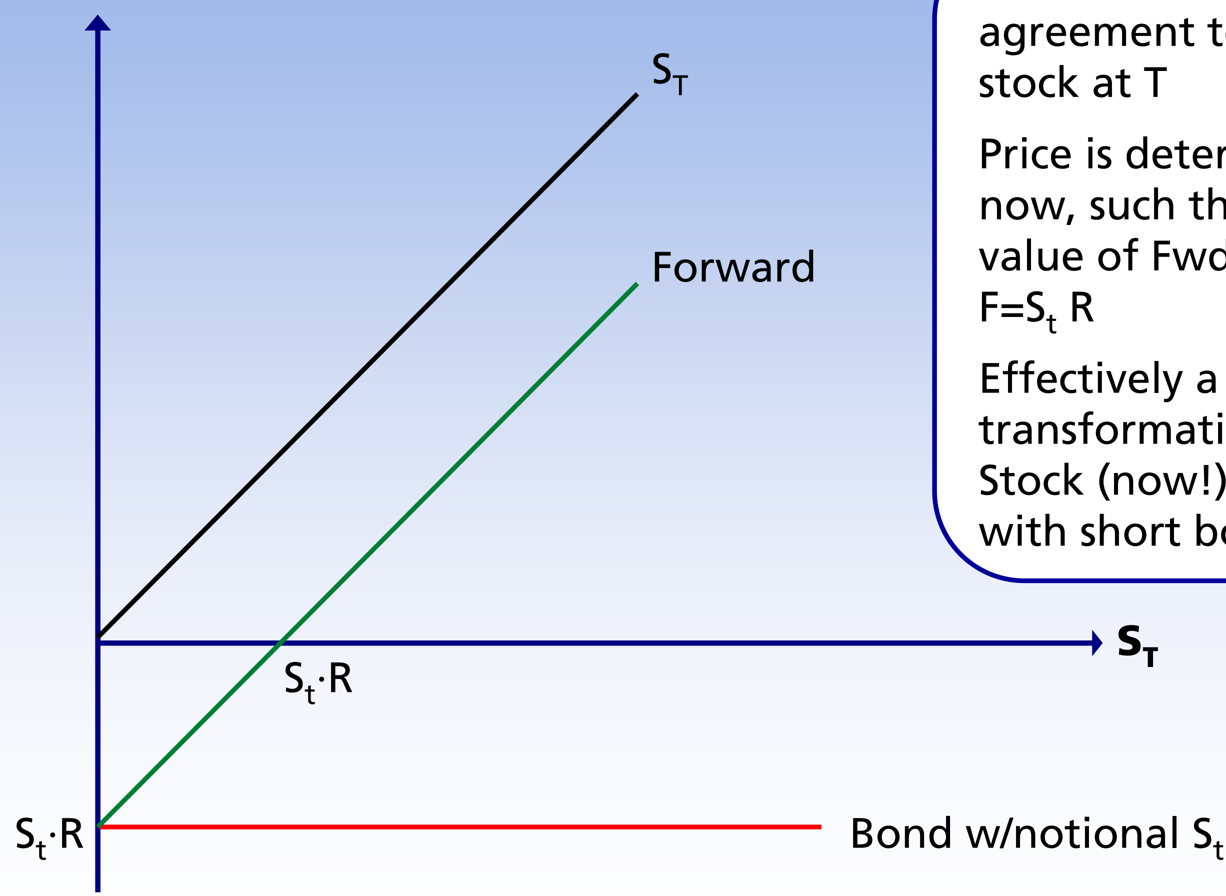


Henceforth: Suppose the stock pays no dividend until T

A static strategy of stock and bond replicates a forward contract on the stock

FORWARD PRICE OF STOCK

Payoffs



Forward on stock is agreement to buy stock at T

Price is determined now, such that current value of Fwd is zero $F = S_t R$

Effectively a liquidity transformation: Long Stock (now!) financed with short bond

As opposed to forwards, futures are standardized and exchange-traded which reduces settlement risks

FORWARD VERSUS FUTURES

BACKUP

Forward	Futures
Traded on over-the-counter market	Traded on an exchange
Non-standardized	Standardized contract
Usually one specified delivery date	Range of delivery dates
Settled at end of contract	Settled daily
Delivery (or final cash settlement) taking place	Contract is usually closed out prior to maturity

Hull, 3rd ed, Table 2.3

Key about options is that they embody a right, not an obligation

OPTIONS: OVERVIEW

Options give their holder
the right – but not the obligation! –
to purchase (respectively sell) an underlying security
at a later time
at a price fixed which is determined in advance

Options might vary in an unlimited number of ways, for example,

- in terms of their underlying (stock, bond, currency, index ...)
- in terms of maturity and whether exercise is only at or also prior to maturity
- how the purchase price is set (fixed number, function of past prices ...) or more broadly: how payoffs are defined

Here: Stock option, with a fixed purchase price and a single exercise date

Pricing of options is complicated because it involves dynamic replication over time. Here we will look only at static strategies

Options are all around us

OPTIONS ACTUALLY

BACKUP

Situation	Option
Theater / Bus / Train	Subscribe or not? Subscription *forgoes* the option to decide every time anew. Premium incurred through higher prices for single tickets
Holidays	Cancellation fee / insurance. In former case, premium paid only at exercise, in latter up-front.
Consumer Credit	Protection rights allow consumer to repay loan at any point in time w/o any particular reason. Effectively, she has always option to cancel loan & refinance when interest rates fall. Banks are short an interest rate put option
Monetary Policy	Rules versus Discretion: By retaining the option to conduct discretionary policy, bank / economy pay a premium in terms of worse economic performance

Again, a little glossary might be useful

OPTIONAL TALK

GLOSSARY

• **Call**	Option giving right to purchase
• **Put**	Option giving right to sell
• **Strike**	Exercise price at which underlying can be bought
• **Premium**	Price of an option
• **Maturity**	Time until / at which exercise possible
• **European**	Option which can only exercised at maturity, but not before
• **American**	Option which can be exercised also prior to maturity
• **Binary**	Option paying a fixed amount if an event occurs (e.g. 1 CHF if UBS higher than 105 in three months)
• **Intrinsic Value**	Payoff if you could exercise today
• **In / at / out of the money**	Intrinsic value positive / zero / negative
• **Plain Vanilla**	Standard contract, as opposed to exotic

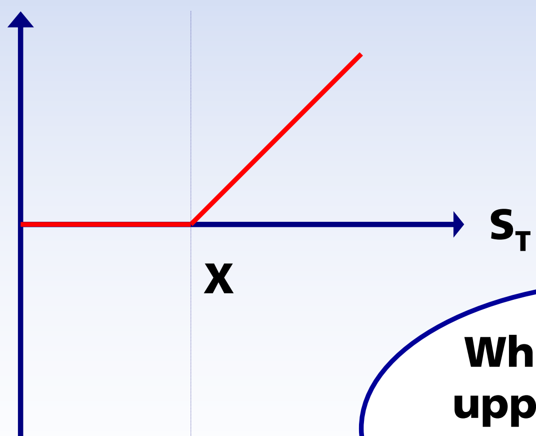
Calls and Puts are only exercised when it is profitable

PLAIN VANILLA OPTIONS

Call

Option to **purchase** stock at price X, exercise if $S_T > X$

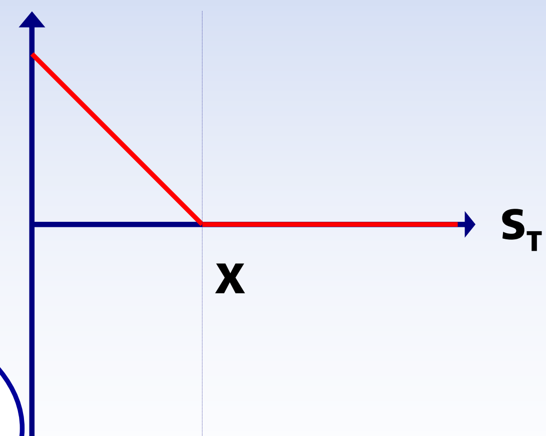
Payoff: max(S_T-X,0)



Put

Option to **sell** stock at price X, exercise if $S_T < X$

Payoff: max(X- S_T,0)

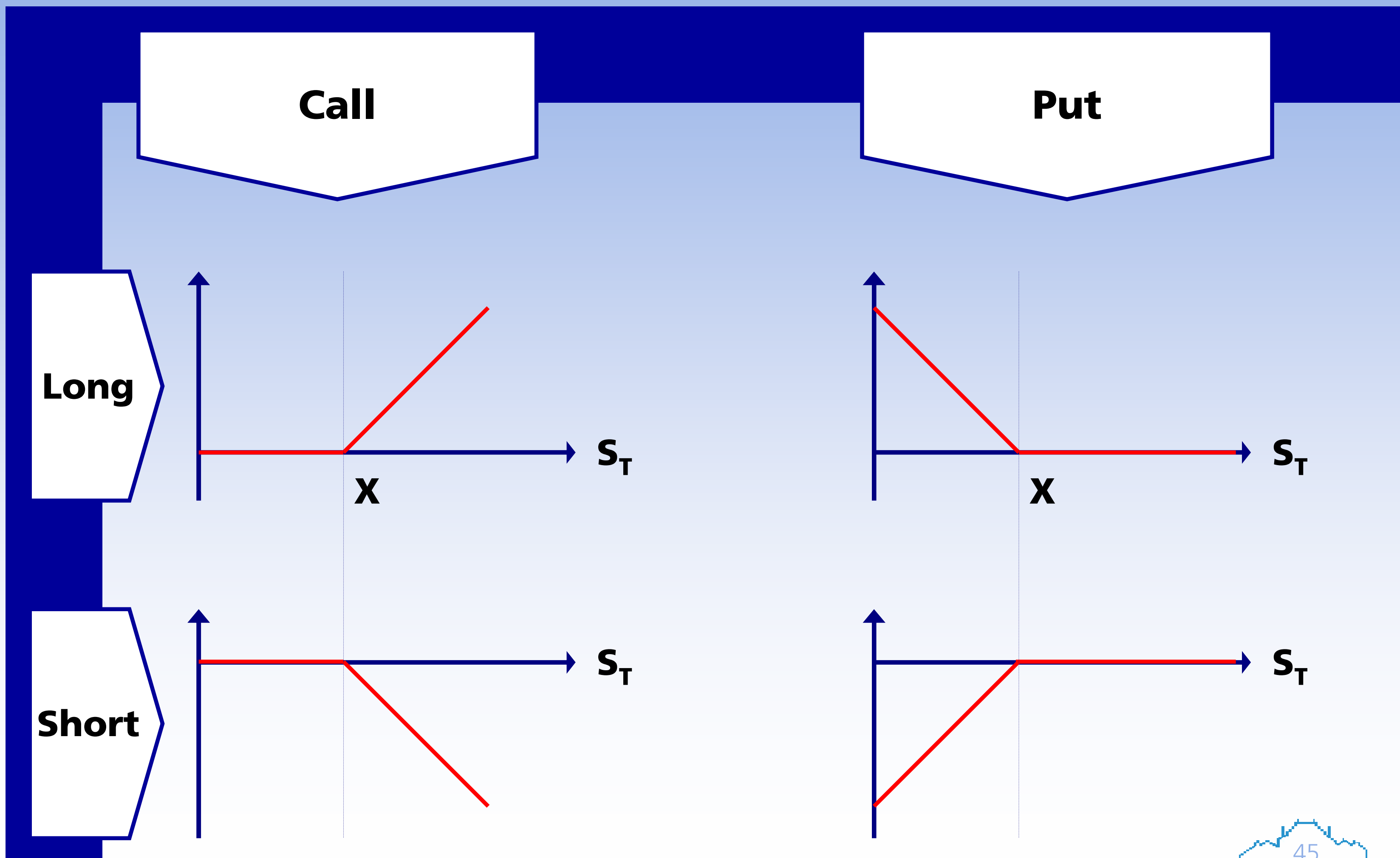


What are lower/ upper bounds on call and put prices?

Derivatives are „zero sum games“, since payoffs on long and short positions cancel

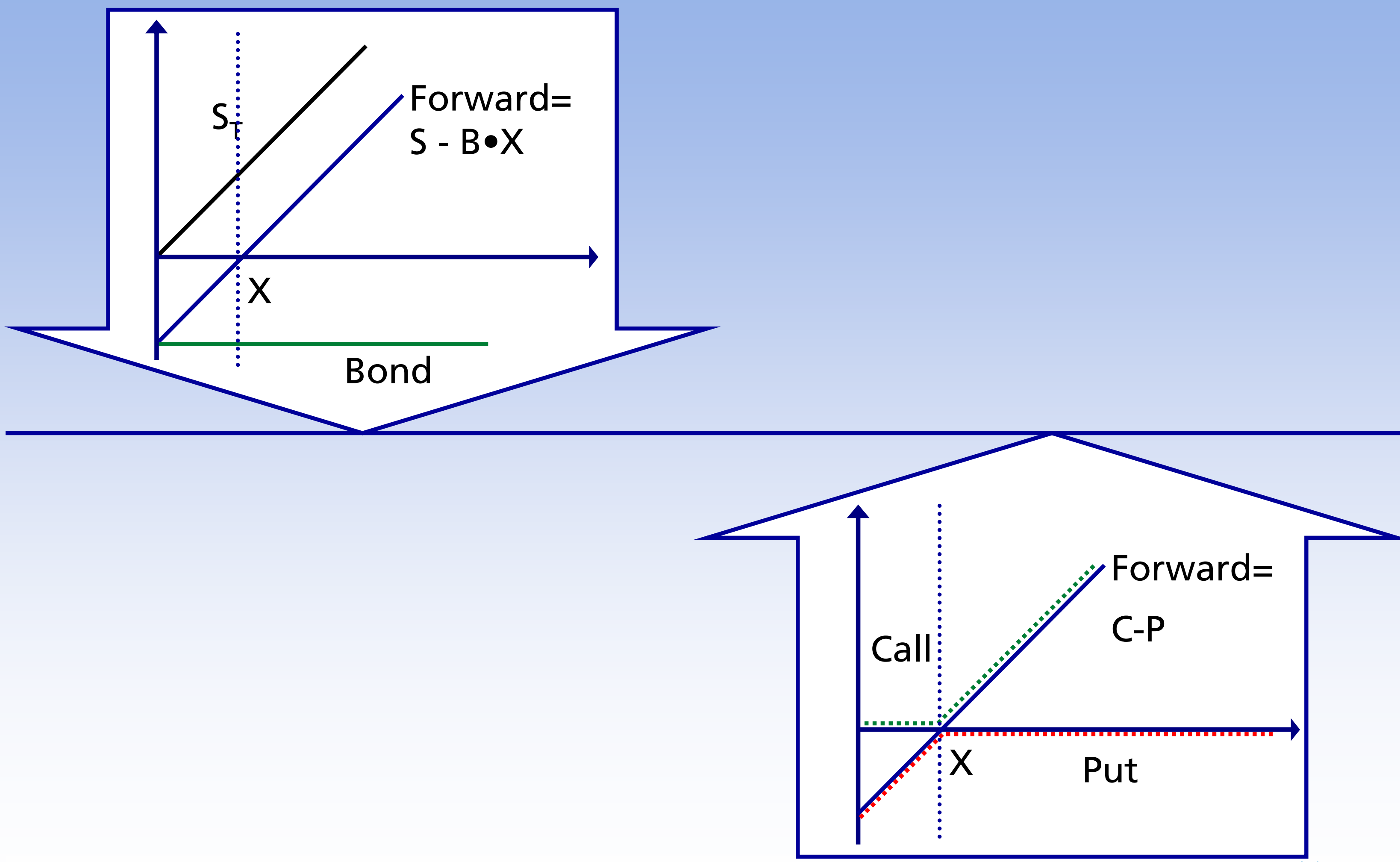
ZERO SUM POSITIONS

BACKUP



Put-Call Parity relates Options to Forwards independently of any pricing model

PUT-CALL PARITY

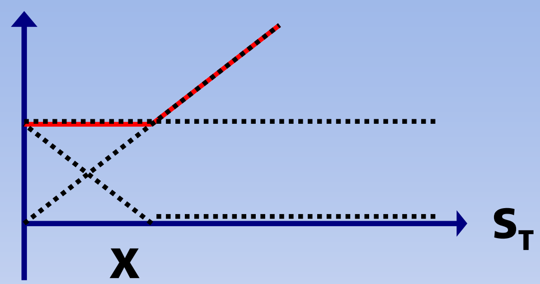


With simple put/call strategies we can engineer various interesting payoff profiles

OPTION STRATEGIES

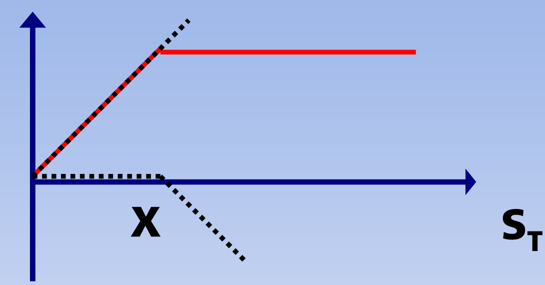
BACKUP

Protective Put



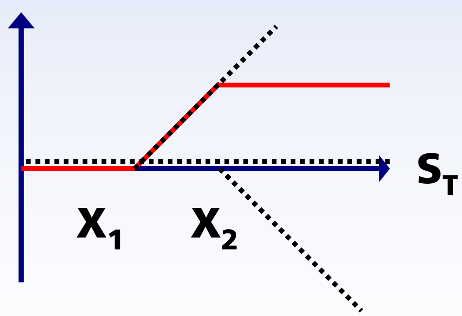
Put, stock and bond (all long)

Covered Call



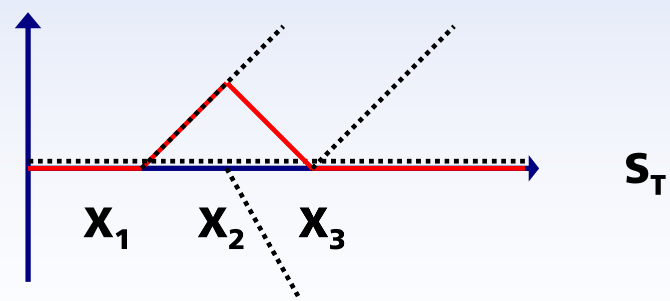
Call short and long stock

Bull Spread:
$X_1 \rightarrow X_2$ gives Binary Call



Call X_1 long and Call X_2 short

Butterfly Spread:
$X_1, X_3 \rightarrow X_2$ gives State Price!



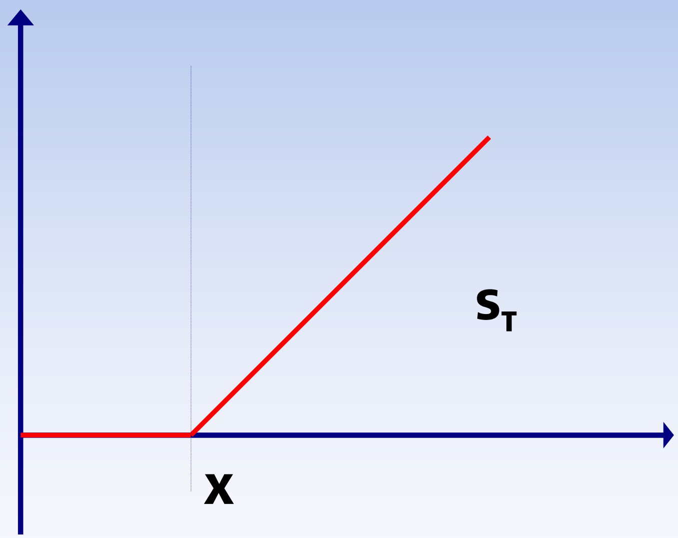
Three Calls: 1 x X_1 long, 2 x X_2 short and 1 x X_3 long

Here we always focused on payoffs. To evaluate success of a strategy, you need to account for the inital price as well

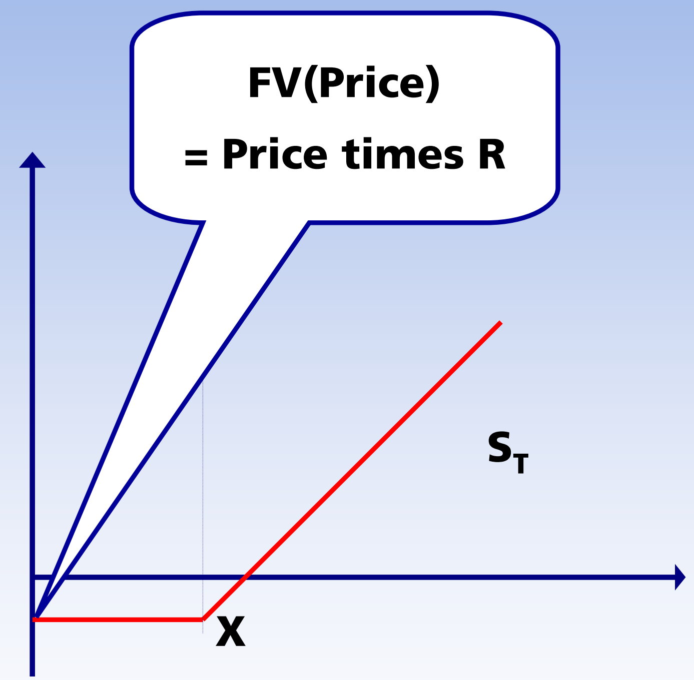
PAYOFF VERSUS PROFIT

BACKUP

Payoff



Profit



We can understand the essence of option pricing with a binomial tree

BINOMIAL SETTING

BACKUP

Given:

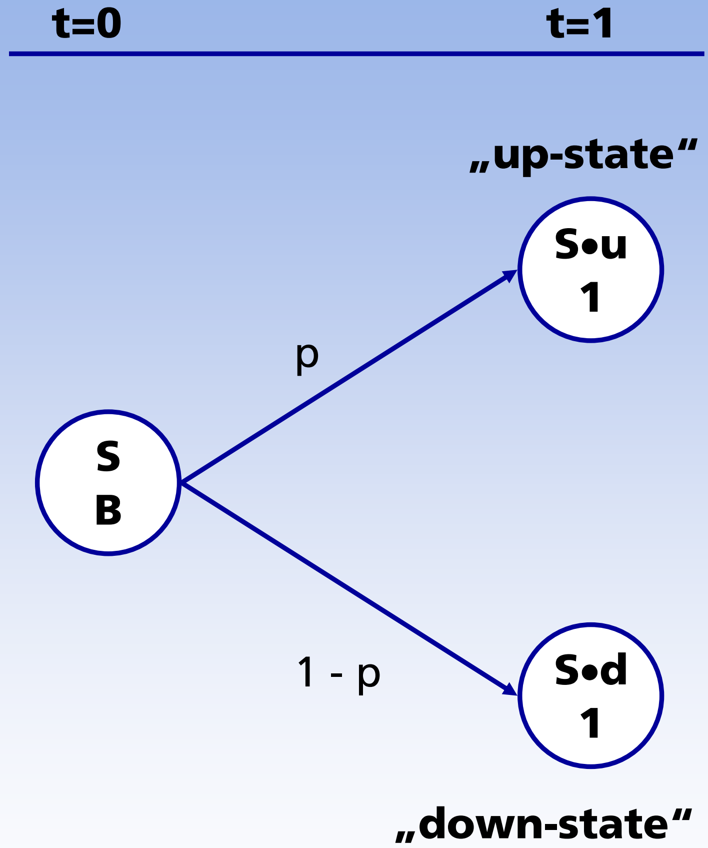
- Stock and bond price given
- Stock can only move up or down (factors u and d which are known)
- probability of up-state is "p"

Example:

S=100, B=1/1.1, u=1.20, d=0.80

Used a lot in practice: Extend it by appending another tree at each node and keep extending …

The infinite limit is the venerable Black-Scholes-Merton Model



Binomial pricing goes back to the ease of menu pricing

BINOMIAL PRICING

BACKUP

Figure out prices of a unit payoff in each state: p_u p_d : **„state prices"**

NA implies for stock and bond:

$$S = S \cdot u \cdot p_u + S \cdot d \cdot p_d$$

$$B = \frac{1}{R} = p_u + p_d$$

Remember:
Every asset is a meal made of payments in up-state and down-state.
Prices exclude happy meal!

The two state prices are pinned down by the two equations for bond and stock

Solving 2 equations in 2 unknowns:

$$p_u = \frac{1}{R}\frac{R-d}{u-d}$$

$$p_d = \frac{1}{R} - p_u$$

(w/o rocket-science)

Probabilities „p" and exp. return **irrelevant**

Spread/volatility u-d and riskfree rate R are key

Example:

Call, X=100

C_u = 20 C_d = 0

C = 20 pu

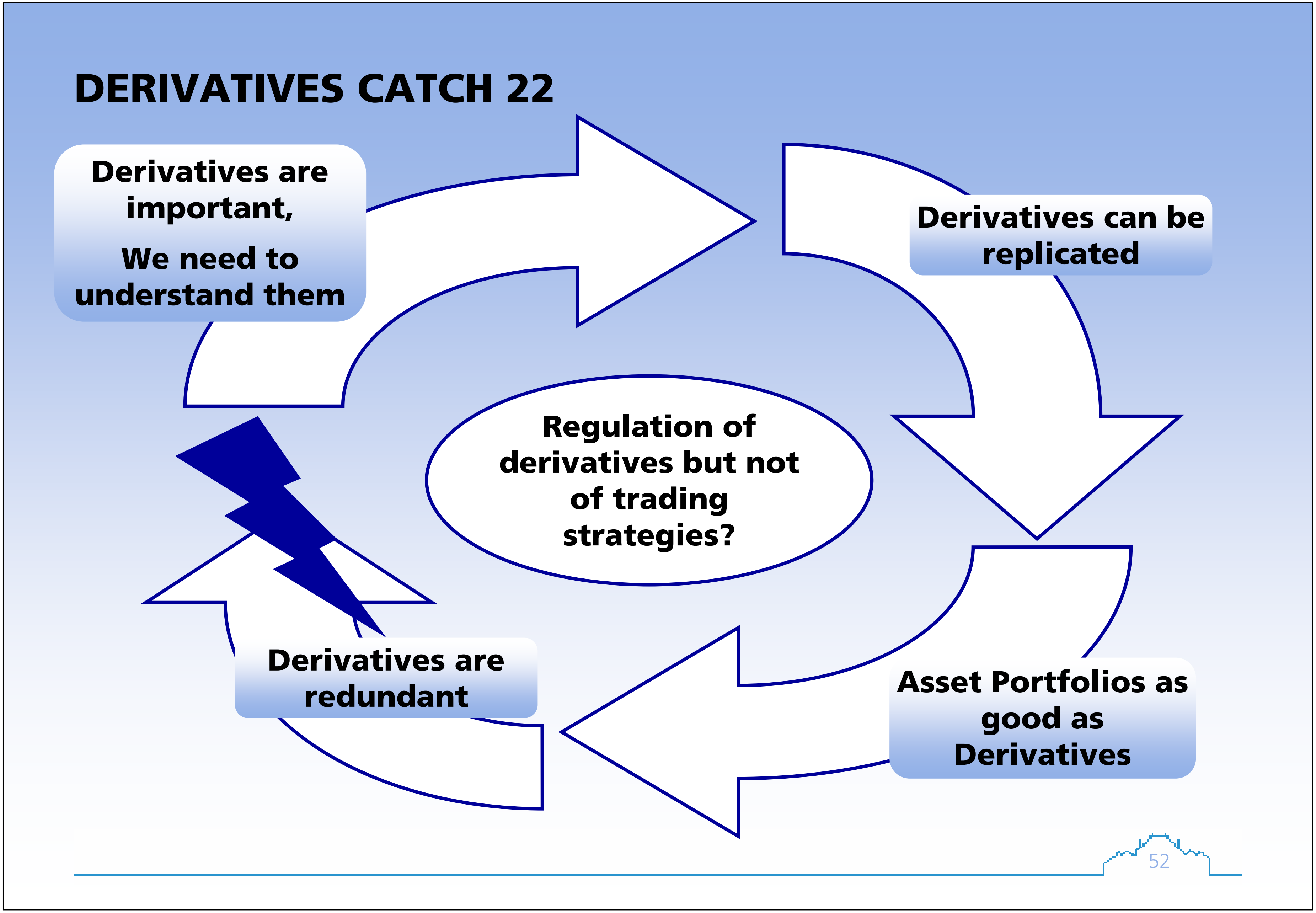
= 20 • ¾ •0.9

= 13.50

AGENDA

Epilog

There must be more to derivatives than just replication



The Crash of 1987 highlighted a crucial difference between replication strategies and contracts

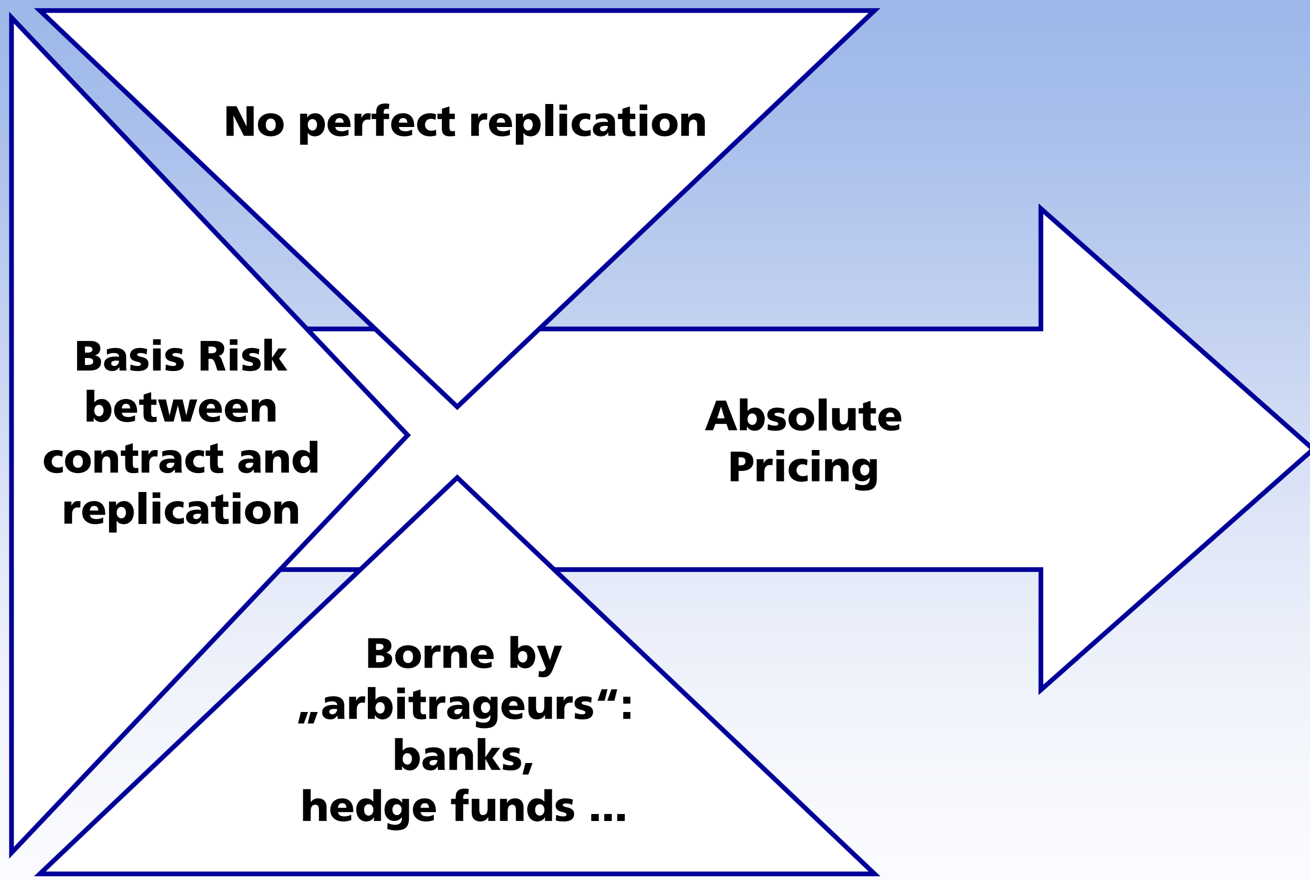
PORTFOLIO INSURANCE ANNO 1987

BACKUP

Put Option Contract	**Replication / Homemade Put**
Buy contract from bank (good counterparty)	Short stock whenever market falls, invest proceeds in Bond
Market Crash: Put matures deep in the money. Collect payoff from counterparty	Market Crash: No or delayed execution. Price jumps impede replication

Limits to replication require absolute pricing

BASIS RISK



This is for another class.

Enjoy the weekend!